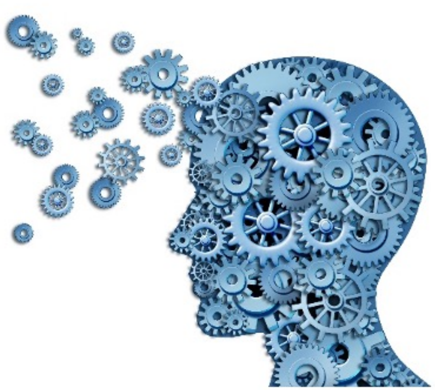
Kettlethorpe
HIGH SCHOOL

MATHS

Year 9 | Delta

Name:

Set:



Unit	Topic	Complete
1	Calculations	
2	Indices and roots	
3	Factors, multiples and primes	
4	Standard form and surds	
5	Algebra basics	
6	Linear equations	
7	Solving quadratic and simultaneous equations	
8	Sequences	
9	Transformations	
10	Accuracy and bounds	
11	Fractions	
12	Probability	
13	Ratio and proportion	
14	Direct and inverse proportion	
15	Polygons and angles	
16	Pythagoras and trigonometry	

Delta Unit 1 : Calculations

Prior Knowledge

- Multiply decimals, without a calculator e.g. 0.4 x 0.6 = 0.24
- Divide decimals without a calculator e.g. 4.5 ÷ 0.9 = 5
- Round to a given number of decimal places e.g. 4.56 to 1.d.p. is 4.6
- Round to a given number of significant figures e.g. 567 to 2 significant figures is 570

Dividing Decimals

Use your short division to divide:

E.G.
32.7 ÷ 3 = 1.09

	1	•	0	9
3	3	•	2	7

4.65 ÷ 0.5 = 46.5 ÷ 5 = 9.3

	0	9	•	3
5	4	6	•	5

Multiplying Decimals

Use the column method – multiply without the decimal points - estimate first for place value.

E.G.
2.8 x 9.4
This is roughly 3 x 9 = 27

		2	8
	x	9	4
	1	1	2
2	5	2	0
2	6	3	2

So to make 2632 near to 27 we make the answer 26.32.

Estimation and approximation

A calculation can be estimated by approximating (rounding) the values within it before calculating.

E.G.

1) Estimate 23 x 56 ≈ 20 x 60 = 1200

2) Estimate $\frac{4.6 \times 21.4}{9.5} \approx \frac{5 \times 20}{10} = \frac{100}{10} = 10$

Product Rule for Counting

To find the total number of outcomes for two or more events, multiply the number of outcomes for each event together.

E.G.
A restaurant menu offers 4 starters, 7 main courses and 3 different desserts.
How many different three-course meals can be selected from the menu?

Multiplying together the number of choices for each course gives 4 x 7 x 3 = 84 different three-course meals.

Literacy

Write the definition of estimate

Use the word estimate within a sentence

Reasoning

There are 30 students in a class, explain using full sentences why there are 30 x 29 = 870 combinations, but only 435 different combinations.

Fluency

Calculate the following without a calculator

1) 23.2 x 45

2) 5.8 x 3.2

3) 34.2 ÷ 0.3

Estimate the following calculations

1) 34 x 56

2) 74 x 321

3) $\frac{453 \times 78}{47.32}$

A restaurant offers 4 starters, 6 mains and 2 deserts how many options are there for a 3 course meal?

Problem Solving

1) 24,532 attended a football match, each paying £11.20, estimate how much was raised

2) A restaurant offers 4 starters, 6 mains and 2 deserts how many options are there for a 2 course meal?

Delta Unit 2 : Indices and roots

Prior Knowledge

Use indices.
E.G. 3^3 =3 x 3 = 9
5^3 = 5 x 5 x 5 =125

Understand what square root means.
E.G. $\sqrt{49} = 7$,
$\sqrt{57} = 7.54 ...$

Use a calculator for indices and roots.

Remember basic index facts.
E.G. $a^0 = 1$

Use BIDMAS for order of operations.
E.G. 5 + 5 x 2 = 15

Use the first 3 index laws.

$a^m \times a^n = a^{m+n}$

$a^m \div a^n = a^{m-n}$

$(a^m)^n = a^{m \times n}$

Negative Indices

A negative power is referred to as taking the ***reciprocal*** of a number, $a^{-m} = \frac{1}{a^m}$

E.G.

$$2^{-3} = \frac{1}{2^3} = \frac{1}{8}$$

Extra: if the number is already a fraction then the reciprocal means the numerator becomes the denominator and vice versa.

E.G. $(\frac{3}{5})^{-4} = \left(\frac{5}{3}\right)^4 = \frac{5^4}{3^4} = \frac{625}{81}$

Fractional Indices

A fractional power means that you take the denominator as a root, $a^{\frac{1}{m}} = \sqrt[m]{a}$

E.G.

$$64^{\frac{1}{3}} = \sqrt[3]{64} = 4$$

Extra: if the fraction has a numerator which is not 1 then we raise the result of our root to that power $a^{\frac{n}{m}} = (\sqrt[m]{a})^n$.

E.G. $125^{\frac{2}{3}} = (\sqrt[3]{125})^2 = 5^2 = 25$

Literacy

Write the definition of indices

Use the word indices within a sentence

Problem Solving

Write the following as single power of 2

$$4^3 \times 2^7$$

Fluency

Calculate the following without a calculator

1) 4^{-2}

2) 3^{-3}

3) 9^{-1}

4) $(\frac{3}{4})^{-2}$

5) $49^{\frac{1}{2}}$

6) $125^{\frac{1}{3}}$

7) $9^{\frac{3}{2}}$

8) $\left(\frac{25}{16}\right)^{\frac{3}{2}}$

Write the following as a single power of 3

$3^3 \times 3^7$

$(3^2)^6$

$\frac{3^9}{3^3}$

$\frac{3^4 \times 3^7}{3^2}$

Reasoning

Barry has answered the following questions, explain what Barry has done wrong in each question

1) $7^{-2} = -49$

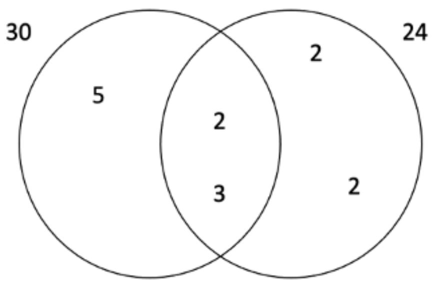
2) $16^{\frac{1}{2}} = 8$

Delta Unit 3 : Factors, Multiples and Primes

Prior Knowledge

Write as the product of prime factors.
E.G.
24 = 2 x 2 x 2 x 3 = 2^3 x 3

Find the HCF and LCM of a given pair of numbers using a Venn Diagram.
E.G. Find the HCF and LCM of 30 and 24.



HCF = Middle = 2 x 3 = 6.

LCM = Multiply all numbers = 2 x 2 x 2 x 3 x 5 = 120

Square roots

You can use prime factors to square root or cube root a number.

E.G.
Find the square root of 484. 484 as a product of prime factors is 484 = 2 x 11 x 2 x 11. So the square root of 484 is 2 x 11 =22

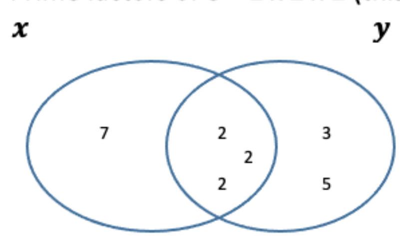
Application of HCF and LCM

Worded
Doughnuts are sold in packs of 8. Cakes are sold in packs of 14. I want to but the same number of cakes as doughnuts. How many packs of each should I buy?

This is a LCM question as both need to appear at the same time LCM of 8 and 14 is 56. (This is not the answer though, I want to know the number of packs for each). I need 7 packs of Doughnuts and 4 packs of cakes.

Reverse
X and Y have an LCM of 840 and HCF of 8.
Neither of X or Y are 8. What could the two numbers be?

1) Prime factors of 840 = 2 x 2 x 2 x 5 x 3 x 7 (these are the numbers that must be in the Venn diagram)
2) Prime factors of 8 = 2 x 2 x 2 (this is the intersection of the Venn diagram)



Each circle represents x or y, so possible answers are x = 7 x 2 x 2 x 2 = 56 and y = 2 x 2 x 2 x 3 x 5 = 120

Literacy

Write the definition of Factor

Use the word factor within a sentence

Reasoning

James is asked to find the lowest common multiple of 85 and 93 he says the answers is 340, explain how you know he must be incorrect.

Fluency

Write the following numbers as a product of their primes.

1) 50

2) 340

3) 270

Find the Highest Common Factor and Lowest Common Multiple of 340 and 270

Problem Solving

1) Use prime factorisation to find the square root of 324, you must show your working.

2) A Barry can but small, medium and large screws, small screws come in bags of 5, medium screws come in bags of 6 and large screws come in bags of 4. If Barry wants the same amount of each screw. How many bags of each should he buy?

Delta Unit 4: Standard form and Surds

Prior Knowledge

- Know the square numbers up to 15^2.
- Convert large values between ordinary and standard form.
  E.G.
  $380000 = 3.8 \times 10^5$
  $7.93 \times 10^6 = 7930000$
- Convert small values between ordinary and standard form.
  E.G.
  $0.00071 = 7.1 \times 10^{-4}$
  $9.32 \times 10^{-2} = 0.0932$

Calculate with Numbers in Standard Form

When adding and subtracting standard form numbers, an easy way is to convert the numbers from standard form into decimal form or ordinary numbers, complete the calculation, convert the answer back into standard form.

E.G.

Calculate $4.5 \times 10^4 + 6.45 \times 10^6$

$= 45{,}000 + 6{,}450{,}000 = 6{,}495{,}000 = 6.495 \times 10^6$

When multiplying and dividing you can use the Laws of Indices. First multiply or divide the first numbers, second apply the Laws of Indices to the powers of 10, third make sure your answer is in standard form.

E.G.

Calculate $(3 \times 10^3) \times (5 \times 10^9)$

$3 \times 5 = 15$, $10^3 \times 10^9 = 10^{12}$. Meaning $(3 \times 10^3) \times (5 \times 10^9) = 15 \times 10^{12} = 1.5 \times 10^{13}$

Rationalise the Denominator

Rationalise the denominator is to remove the surd element from the denominator, it is done by multiplying by the surd.

e.g. (i) $\frac{4}{\sqrt{2}} = \frac{4}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$

$= \frac{4\sqrt{2}}{2}$

$= 2\sqrt{2}$

Simplifying expressions involving surds

Here are some general rules with surds:

$\sqrt{a} \times \sqrt{a} = a$ $\quad$ $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$ $\quad$ $\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$

E.G.

1) $\sqrt{32} = \sqrt{16} \times \sqrt{2} = 4\sqrt{2}$

2) $\sqrt{2}(\sqrt{3} + 5) = \sqrt{6} + 5\sqrt{2}$

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MATHS

Year 9 | Half-term 1: *Unit 4 Standard Form and Surds*

Literacy

Write the definition of rational

Use the word rational within a sentence

Reasoning

Give a reason as to why we would prefer using surds instead of decimals.

Fluency

Calculate the following without a calculator, leave your answer in standard form

1) $(5.4 \times 10^4) + (3.1 \times 10^3)$ 2) $(6 \times 10^3) \times (2 \times 10^4)$ 3) $(1.2 \times 10^{14}) \div (2 \times 10^5)$

Simplify the following to their simplest form

1) $\sqrt{196}$ 2) $\sqrt{50}$ 3) $\sqrt{3} \times \sqrt{6}$ 4) $\sqrt{15} \times \sqrt{20}$

Rationalise

1) $\frac{6}{\sqrt{3}}$ 2) $\frac{\sqrt{3}}{2\sqrt{5}}$ 3) $\frac{2\sqrt{2}}{\sqrt{18}}$

Problem Solving

1) The distance from the sun to the Earth is 1.5×10^8 km from the sun. Venus is 4×10^7 km closer to the sun than the Earth. How close to the sun is Venus?

2) Mercury is 5.8×10^7km from the sun, Pluto is 100 times further away than Mercury. How far away from the sun is Mercury?

Delta Unit 5: Algebra

Prior Knowledge

Simplify an algebraic expression.

Write an algebraic expression.

Substitute numbers into expressions.

Substitute numbers into formulae.

Write a formula from a description.

Expand expressions involving single brackets.

Factorise a linear algebraic expression.

Expand expressions involving double brackets.

Expressions, Equations, Formulas, and Identities

Expression: An expression is a collection of terms combined using the operations +, -, × or ÷ for example $3x + 2,\ xy + 7b\ \ or\ \ x^2 - 3x + 7$

Equations: An equation states that 2 expressions are equal and involves 1 unknown, e.g. $4b - 2\ =\ 6$

Formulas: A formula is an equation which shows the rule which connects more than one variable e.g. $A\ =\ l \times W,\quad y = mx + c$

Identity: An identity is a statement that is true no matter what values are chosen e.g. $4a^2 \times a = 4a^3$

Factorising a quadratic expression

A quadratic expression is one which involves a quadratic term e.g. $x^2 + 9x + 20$

E.G.
Factorise the expression $x^2 + 9x + 20$ You are looking for the 2 numbers which multiply to make 20 and add to make 9, these are 4 and 5. These are the factors and give the answer:

$$x^2 + 9x + 20 = (x + 4)(x + 5)$$

E.G.
Factorise the expression $x^2 - 5x - 24$ You are looking for the 2 numbers which multiply to make -24 and add to make -5, these are -8 and 3. These are the factors and give the answer:

$$x^2 - 5x - 24 = (x - 8)(x + 3)$$

Hint: Expand the double brackets to check you have the correct answer.

Literacy

Write the definition of an identity.

Use the word identity within a sentence

Reasoning

Bob says when you expand $(d - 6)^2$ you get $d^2 + 12d + 36$. Explain what mistake Bob has made.

Fluency

Substitute a = 5 and b = -3 into these expressions.

1) 3a + 7b
2) 6a – 4b
3) $3a^2$

Expand and simplify

1) 3(x + 4) - 2(x - 5)
2) (3x +2)(x – 4)
3) (2x + 4)(x – 5)

Factorise these expressions

1) $9x^2 - 6x$
2) $x^2 - 5x + 4$
3) $x^2 - 25$

Problem Solving

1) When $(x + a)(x - 5)$ is expanded you get $x^2 + bx - 10$.
Find a and b.

2) Find the values of a, b and c to complete this factorization.
$6y^2 - 17y - a = (by - 10)(2y + c)$

Delta Unit 6: Equations

Prior Knowledge

Substitute numbers into formulae.

Write a formula from a description.

Write and solve a two-step equation.

Write and solve equations involving brackets.

Write and solve equations with unknowns on both sides.

Changing the subject of a formula

A formula gives a rule for calculating one unknown, you can rearrange this to make x the 'subject', this means you want the formula to start with x.

E.G.

$c = \frac{5(f-32)}{9}$

x 9 x 9

$9c = 5(f-32)$

÷ 5 ÷ 5

$\frac{9}{5}c = f - 32$

+ 32 + 32

$\frac{9}{5}c + 32 = f$

Iteration

Iteration means repeatedly carrying out a process. To solve an equation using iteration, start with an initial value and substitute this into the iteration formula to obtain a new value, then use the new value for the next substitution, and so on.

In order to start the process you need an iterative formula derived from the equation

E.G.
Find the solution to the equation $x^3 + 5x = 20$ using the initial value $x_0 = 2$, and the iterative formula $x_{n+1} = \sqrt[3]{20 - 5x_n}$, giving the answer to 3.d.p.

$x_1 = \sqrt[3]{20 - 5 \times 2} = \sqrt[3]{10} = 2.154...$

Substituting iteratively gives:

$x_2 = \sqrt[3]{20 - 5 \times 2.154...} = \sqrt[3]{9.227...} = 2.097(3dp)$

$x_3 = \sqrt[3]{20 - 5 \times 2.097...} = \sqrt[3]{9.512...} = 2.118(3dp)$

$x_4 = \sqrt[3]{20 - 5 \times 2.118...} = \sqrt[3]{9.405...} = 2.111(3dp)$

$x_5 = \sqrt[3]{20 - 5 \times 2.111...} = \sqrt[3]{9.445...} = 2.114(3dp)$

$x_6 = \sqrt[3]{20 - 5 \times 2.114...} = \sqrt[3]{9.430...} = 2.113(3dp)$

$x_7 = \sqrt[3]{20 - 5 \times 2.113...} = \sqrt[3]{9.436...} = 2.113(3dp)$

Since x_6 and x_7 give the same value to 3.d.p. (2.113) then the solution to $x^3 + 5x = 20$ to 3 d.p. is 2.113

Solving equations with the unknown on both sides

E.G.

5x – 4 = 2x + 20
-2x -2x
3x – 4 = 20
+4 +4
3x = 24
÷3 ÷3
x = 8

Solving equations with brackets

Expand the bracket then use inverse operations to balance the equation.

E.G. 2(3k + 4) = 46
6k + 4 = 46
-4 -4
6k = 42
÷6 ÷6
k = 7

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MATHS

Year 9 | Half-term 2: *Unit 6 Equations*

Literacy

Explain what iteration means.

Explain why we use iteration.

Reasoning

Charlie was asked to rearrange the formula to make a the subject.
Explain what mistake Charlie has made.

$3a^2 + \mathrm{b} = \mathrm{c}$

$3a^2 = \mathrm{c} - \mathrm{b}$

$3a = \sqrt{c - b}$

$a = \frac{\sqrt{c - b}}{3}$

Fluency

Solve

1) $\frac{2x - 4}{3} = 2$

2) $7x + 12 = 3\mathrm{x} - 8$

3) $7(2x + 3) = 28$

Make t the subject

1) $v = u + at$

2) $5t + 7 = 7t - b$

3) $at - b = ct + d$

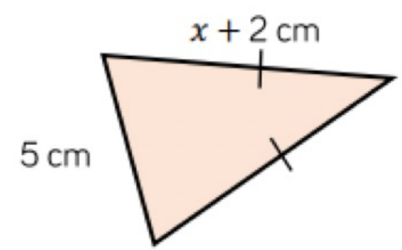
Starting with $x_0 = 0$, use each iteration formula three times to find an estimate for its solution.
Give your answer correct to 3 decimal places.

1) $x_{n+1} = \frac{1}{4} - \frac{x_n^3}{4}$

2) $x_{n+1} = \frac{5}{x_n^2 + 7}$

Problem Solving

The perimeter of this triangle Is 29cm.
Calculate the area.



Delta Unit 7: Solving Quadratic and Simultaneous Equations

Prior Knowledge

Substitute into expressions.

Solve linear equations.

Expand double brackets.

Manipulate expressions.

Factorise quadratics.

Square rooting gives 2 solutions.

Roots are solutions to quadratics.

Solving quadratic equations using the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Solving quadratic equations by factorising

A quadratic will have 2 solutions. Factorising will give us 2 equations to solve.

E.G.
Solve $x^2 + 5x - 24 = 0$

First factorise $x^2 + 5x - 24 = 0$

$$(x + 8)(x - 3) = 0$$

Either bracket can be equal to 0, these are the 2 solutions, so the two solutions are:

$$x + 8 = 0 \quad or \quad x - 3 = 0$$
$$x = -8 \quad or \quad x = 3$$

Completing the square

E.G.
Write $x^2 + 8x - 5$ in completed square form.

First $x^2 + 8x$ can become $(x + 4)^2$however when expanded $(x + 4)^2$ becomes $x^2 + 8x + 16$ we need to counteract the 16 by subtracting it.

So we write:

$$x^2 + 8x - 5 = (x + 4)^2 - 16 - 5$$
$$= (x + 4)^2 - 21$$

Solving simultaneous equations

Simultaneous equations involve 2 variables, and you need a solution for each variable, you eliminate one variable first.

E.G.
Solve the simultaneous equations:

$$3x + 4y = 19$$
$$5x + 6y = 30$$

$$3x + 4y = 19 \quad \times 3 \rightarrow \quad 9x + 12y = 57$$
$$5x + 6y = 30 \quad \times 2 \rightarrow \quad 10x + 12y = 60$$

$$-1x + 0y = -3$$
$$x = 3$$

Substitute $x = 3$ into one equation to find the value of y:

$$3(3) + 4y = 19$$
$$9 + 4y = 19$$
$$4y = 10$$
$$y = 2.5$$

Check $y = 2.5 \quad x = 3$ in the other equation.

$5 \times 3 + 6 \times 2.5 = 15 + 15 = 30$ so is correct.

So our solution is $y = 2.5 \; and \; x = 3$

Hint: When solving graphically, plot both lines and see where they intersect.

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MATHS

Year 9 | Half-term 2: *Unit 7 Solving Quadratic and Simultaneous Equations*

Literacy

Write the definition of Solution

Use the word solution within a sentence

Reasoning

Solve $x^2 - 12 = 24$

Matt writes the following:
$(x - 6)(x - 6) = 0$
$x = 6$ and $x = 6$
Comment on Matt's solution

Fluency

Solve by factorising:

1) $x^2 + 5x - 14 = 0$ 2) $x^2 - 8x = 9$ 3) $3x^2 - 10x - 8 = 0$

Solve by completing the square

1) $x^2 + 6x - 10 = 0$ 2) $x^2 - 4x + 2 = 0$ 3) $x^2 - 8x - 6 = 0$

Solve

1) $3x + 2y = 21$
$2x - 5y = -5$

2) $2x - 5y = 18$
$4x - 3y = 22$

Problem Solving

1) Tim buys 5 apples and 8 bananas; it costs him £2.35. Anya buys 2 apples and 3 bananas, if costs her 91p. Find the cost of 2 apples and 2 bananas.

2) A rectangle has length (x+5)cm and width (x+2)cm. The rectangle has an area of $70cm^2$.
Find the value of x and the perimeter of the rectangle.

Delta Unit 8: Sequences

Prior Knowledge

Substitute numbers into expressions.

Substitute numbers into quadratic expressions.

Recognise simple sequences.

Work out the terms of an arithmetic sequence using the term-to-term rule.

Work out a given term in a simple arithmetic sequence.

Work out and use the nth term for an arithmetic sequence.

Generating a Quadratic Sequence

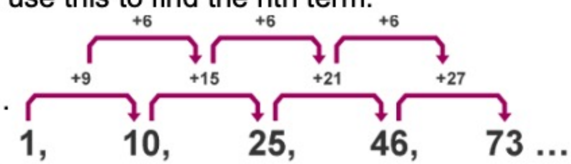
E.G.
Generate the first 4 terms of the sequence $2n^2 – 5$.

$2 \times 1^2 – 5 = -3$, $2 \times 2^2 – 5 = 3$, $2 \times 3^2 – 5 = 13$, $2 \times 4^2 – 5 = 27$ so the answer is -3, 3, 13, 27.

Finding the Nth Term of a Quadratic Sequence

In a quadratic sequence, the difference between each term is different, however the difference between the differences is the same. We use this to find the nth term.

E.G.
Find the n^{th} term of the sequence 1,10, 25, 46, 73,…



First step is to find the difference of the differences.
The differences of the differences is 6, therefore the coefficient of n^2 will be $6 \div 2 = 3$

We generate the sequence $3n^2$ which is 3,12, 27, 48, 75.
Comparing this to our current sequence it is 2 more for each term, therefore the final n^{th} term is $3n^2 - 2$

Finding the common ratio for a Geometric Sequence

A geometric sequence is not generated by adding an amount but by multiplying by the same value each time. The number which you multiply by each time is known as the common ratio.

To find the common ratio you divide two subsequent terms in a geometric sequence.

E.G.

1) 8, 24, 72, 216 in this geometric sequence the common ratio = $\frac{24}{8} = 3$

2) 3,6,12,24 in this geometric sequence the common ratio is $\frac{6}{3} = \frac{12}{6} = 2$

Literacy

Give the definition of a geometric sequence.

Give the definition of a quadratic sequence.

Reasoning

Here are the first four terms of an arithmetic sequence.

3 10 17 24

(a) Find, in terms of *n*, an expression for the *n*th term of this arithmetic sequence.

(b) Is 150 a term of this sequence?

You must explain how you get your answer.

Fluency

Write the first four terms of the sequences:

1) $5n - 3$ 2) $3n^2 + 5$ 3) $2 \times 3^{n-1}$

Find the nth term for each of these sequences.

1) 5 7 9 11 13 2) 12 9 6 3 0 3) -1 3 7 11 15

Problem Solving

Here are the first four terms of a quadratic sequence.

3 8 15 24

(a) Find an expression, in terms of *n*, for the *n*th term of this sequence.

The *n*th term of a different sequence is $2^n + 5$

(b) Show that 36 is **not** a term of this sequence.

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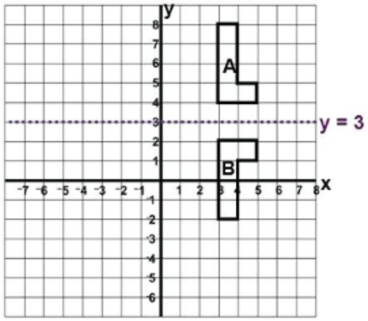
Delta Unit 9: Transformations

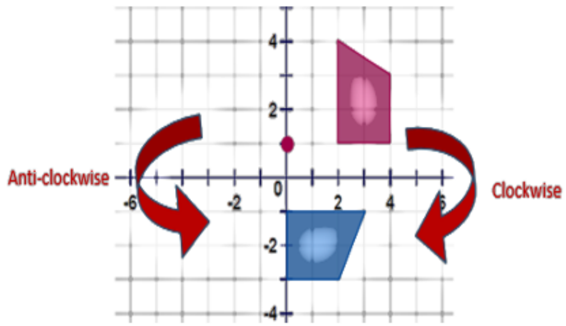
Prior Knowledge

Know the equations of horizontal and vertical lines.

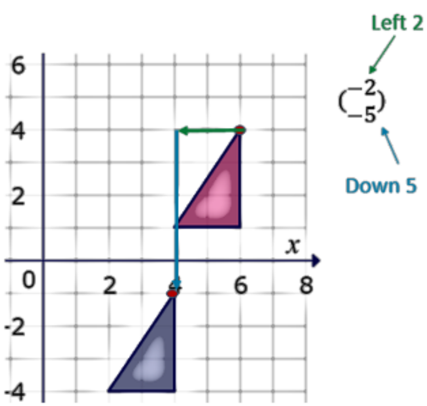
Reflect in a given line of reflection.



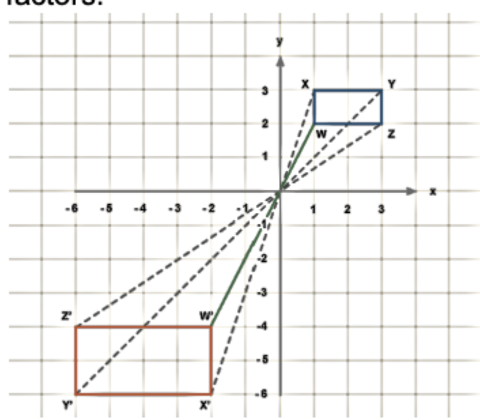
Rotate a shape around a given point.



Translate a shape by a given. vector.



Enlarge a shape by a given scale factor. including negative and fractional scale factors.



Describing transformations

You will be asked for a single transformation, not a combination of two. These are key parts to each:

Rotation:
Direction, centre and degrees of turn.
E.G.
A rotation clockwise of 270^0 centre (3,2).

Reflection:
Line of reflection.
E.G.
A reflection in the line y = 5.

Translation:
Vector for the translation.
E.G.
A translation by the vector $\begin{pmatrix} 5 \\ -4 \end{pmatrix}$.

Enlargement:
Scale factor, centre of enlargement.
E.G.
An enlargement of scale factor 3, centre (3,7).

Invariant points

Invariant points are points on shape, where after a transformation they haven't moved and have remained in the same place.

Literacy

Write the definition of Congruent

Use the word congruent within a sentence

Reasoning

Glenys doesn't understand Invariant points.
Describe or draw one example of invariant points for each of the 4 types of transformation.

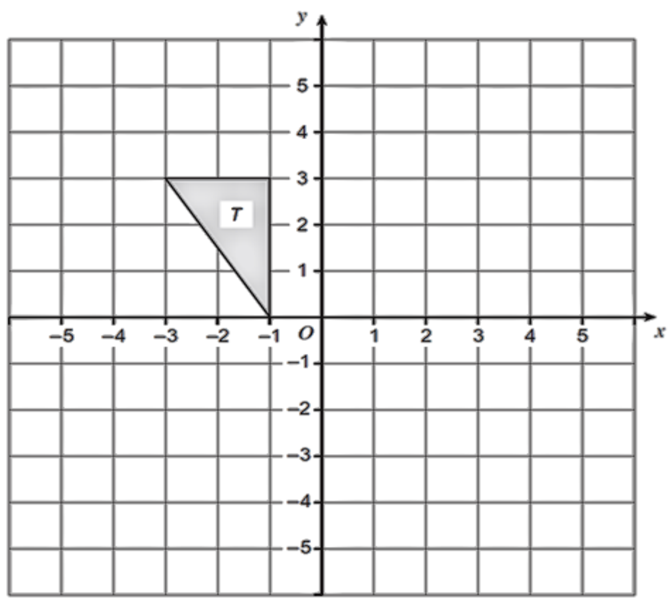
Fluency

Describe what the following translation vectors would do to a shape:

1) $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$ 2) $\begin{pmatrix} 4 \\ -1 \end{pmatrix}$ 3) $\begin{pmatrix} -5 \\ 0 \end{pmatrix}$

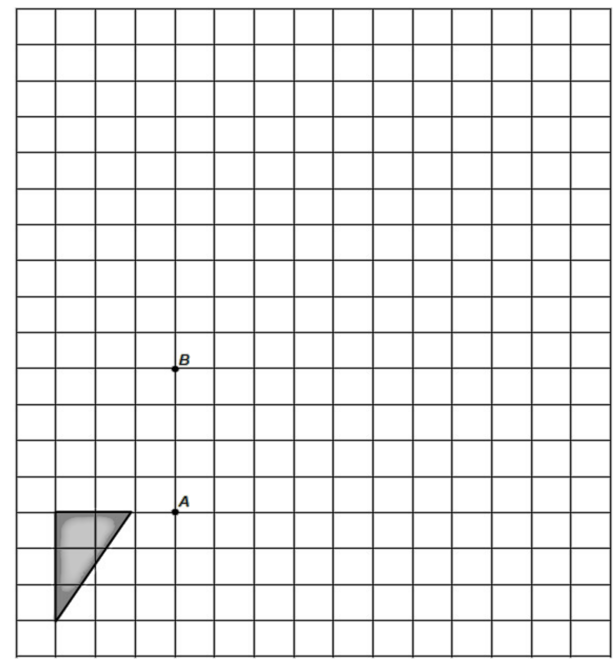
1) Reflect the shape T in the line y = -1

2) Rotate the shape T 180° about the origin.



Problem Solving

Enlarge the shape scale factor -2, centre A.
Rotate the new shape 90° anticlockwise about B.



Delta Unit 10: Accuracy and Bounds

Prior Knowledge

To be able to round to a given degree of accuracy.

Use inequality notation.

Estimate by rounding to 1s.f.

Bounds and Limits

Bounds are the biggest and smallest number a number could have been before it was rounded.

E.G.
The length of a pencil is 5.8cm to the nearest mm. Give the upper and lower bound.

Answer $5.75 < x \leq 5.85$

Calculations with Bounds

When calculation with the rounded numbers A and B, we need to bear in mind the following rules:

UB means Upper Bound
LB means Lower Bound

	MAXIMUM	MINIMUM
A + B	$UB_A + UB_B$	$LB_A + LB_B$
A - B	$UB_A - LB_B$	$LB_A - UB_B$
A x B	$UB_A \times UB_B$	$LB_A \times LB_B$
A ÷ B	$UB_A \div LB_B$	$LB_A \div UB_B$

Accuracy of Bounds

For considering bounds, we calculate the upper and lower bound and see where they coincide to give us an answer.

E.G.
The average fuel consumption (c) of a car in kilometres per litre is given by the formula,

$$c = \frac{d}{f}$$

Where d is distance in kilometres, and f is the fuel used, in litres.

Given d = 163 to 3 significant figures and f =45.3 to 3 significant figures.
By considering bounds give c to a suitable degree of accuracy

Answer
UB(d) = 163.5 UB(f)=45.35
LB(d) = 162.5 LB(f)=45.25

$$UB(c) = \frac{UB(d)}{LB(f)} = \frac{163.5}{45.25} = 3.6132596685$$

$$LB(c) = \frac{LB(d)}{UB(f)} = \frac{162.5}{45.35} = 3.5832414553$$

To the nearest whole they are both 3
To 1 decimal place they are both 3.6
To 2 decimal places they are different 3.58 and 3.61

Meaning that we give an answer of 3.6 as the upper and lower bound agree to 1 decimal place.

Literacy

Write the definition of Estimation

Use the word estimation within a sentence

Reasoning

Mikayla front garden is a square with sides 4m (to the nearest metre). Her back garden is rectangular with sides 3m by 5m (to the nearest metre).
She says "My front garden is definitely bigger". Is she correct?

Fluency

Write the error interval for the following:

1) £34 (to the nearest £1)   2) 4.8 (to 2dp)   3) 280 (to 3sf)

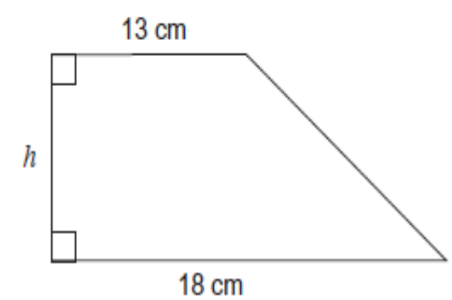
4) $p = 34$ (to 2sf) $q = 8.7$ (to the nearest tenth)

Find the Upper Bound and Lower Bound of the following expressions:

a) $p + q$ b) pq c) q^2 d) $q - p^2$

Problem Solving

The area of the trapezium is 320cm^2 (to 2sf)
The lengths of 13cm and 18cm are given to the nearest integer.
Find the maximum value of *h*.



Delta Unit 11: Fractions

Prior Knowledge

Express a given number as a fraction of another.

Simplify and find equivalent fractions.

Find a fraction of amount.

Convert between improper fractions and mixed numbers.
E.G. $\frac{11}{5} = 2\frac{1}{5}$

Add/subtract fractions by finding a common denominator.
E.G. $\frac{5}{8} + \frac{2}{3} = \frac{15}{24} + \frac{16}{24} = \frac{31}{24} = 1\frac{7}{24}$

Multiply fractions.
E.G. $\frac{3}{7} \times \frac{5}{8} = \frac{15}{56}$

Divide fractions.
E.G. $\frac{5}{7} \div \frac{3}{8} = \frac{5}{7} \times \frac{8}{3} = \frac{40}{21} = 1\frac{19}{21}$

Fractions to recurring decimals

Fractions only terminate if 2 and 5 are the only primes in the prime decomposition of the denominator.

E.G. $\frac{2}{15}$ will recur as the denominator is $15 = 3 \times 5$ as a product of prime factors. There is a 3 present.

Recurring decimals to fractions

Step 1: Write the recurring decimal equal to x.

Step 2: Multiply to find $10x$ and $100x$, you may sometimes need $1000x$. You are trying to get the decimal places to match up.

Step 3: Subtract two of your equations to eliminate the decimal places.

Step 4: Solve the equation to find x. You will need to simplify your fraction.

E.G. Convert $0.\dot{5}\dot{7}$ to a fraction.

$$x = 0.5757575757\ldots$$
$$10x = 5.75757575757\ldots$$
$$100x = 57.5757575757\ldots$$

Notice the decimals for x and $100x$ line up. We subtract to eliminate the recurring decimals.

$$100x = 57.5757575757\ldots$$
$$x = 0.5757575757\ldots$$
$$99x = 57$$
$$x = \frac{57}{99}$$

Literacy

Write the definition of reciprocal.

Reasoning

$\frac{3}{5}$ of members in a tennis club are women , $\frac{3}{8}$ of these women are left handed

Explain why the smallest number of members of the club must be 40.

Fluency

Fractions of amounts

1) Find $\frac{3}{5}$ of 75

2) 2) If $\frac{2}{5}$ of an amount is 14, what is the original amount?

Add/subtract fractions and mixed numbers

1) $\frac{2}{5} + \frac{1}{3}$

2) $3\frac{1}{2} - 2\frac{4}{5}$

Multiply/divide fractions and mixed numbers

1) $\frac{2}{5} \times \frac{3}{4}$

2) $3\frac{4}{7} \div 1\frac{2}{3}$

Problem Solving

Prove algebraically that the recurring decimal $0.1\dot{4}\dot{2}$ can be written as the fraction $\frac{47}{330}$

Delta Unit 12: Probability

Prior Knowledge

Write probabilities as fraction, decimal or percentage.

Calculate probabilities.

Know that probabilities sum to 1.

Identify events that are mutually exclusive.

Work out experimental probabilities and use them to estimate frequency.

Use tree diagrams to calculate probability of independent events.

Use sample space diagrams to calculate probabilities.

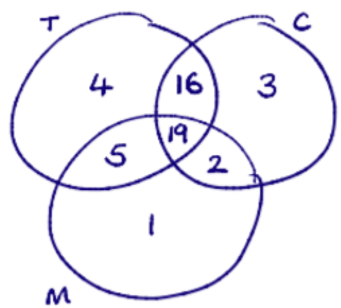
Use Venn Diagrams to calculate probabilities.

Venn Diagram

When drawing a Venn diagram work out from the centre.

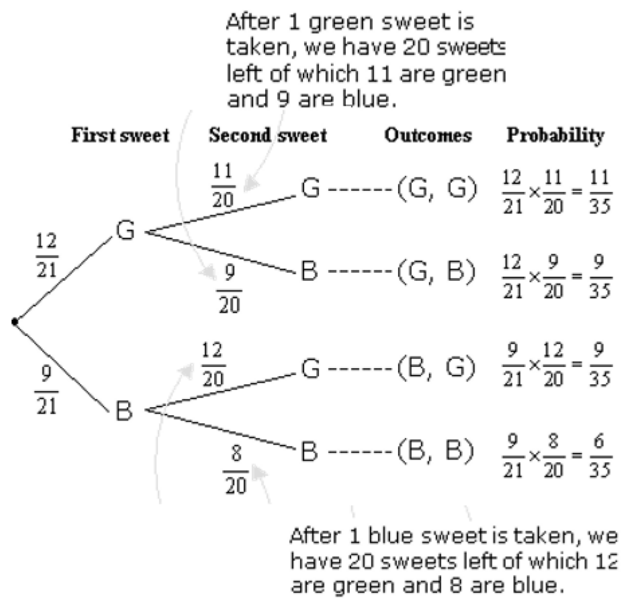
E.G.
Sami asked 50 people which drinks they liked from tea, coffee and milk.

All 50 like at least one of the drinks.
19 people like all three drinks.
16 people like tea and coffee but not milk.
21 people like coffee and milk.
40 people like coffee.
1 person likes only milk.



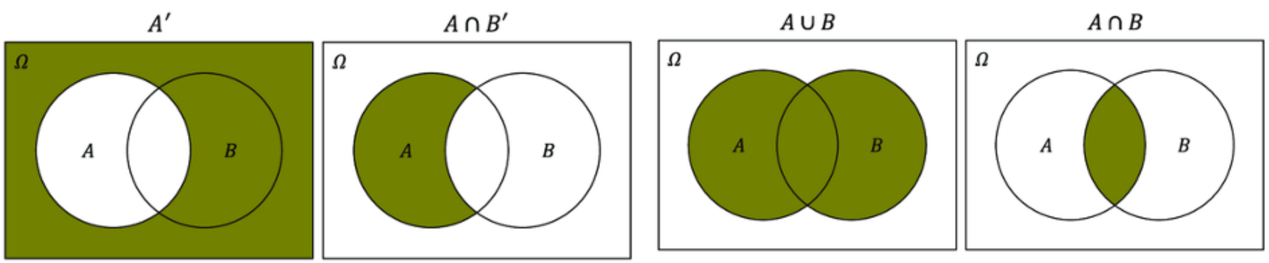
Conditional probability

E.G.
Jack is picking a sweet from the bag. He eats it and then picks another sweet. Find the probability of selecting two green sweets.



P(both green) = $\frac{11}{35}$.

Set Notation



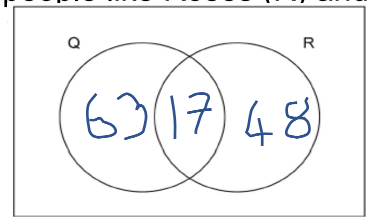
Literacy

Write the definition of Conditional

Use the word conditional within a sentence

Reasoning

120 people take a survey on chocolate preferences. 63 people like Quality Street (Q). 48 people like Roses (R) and 17 people like both.



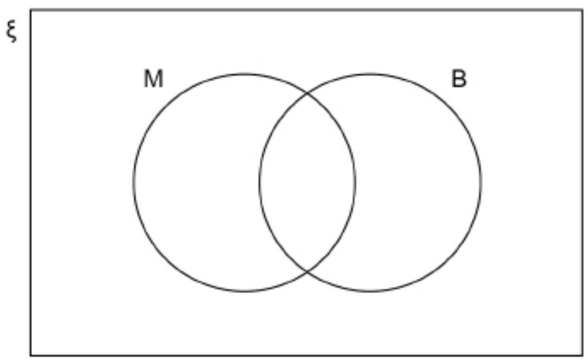
Ron completes the Venn diagram as shown. Comment on his work.

Fluency

The Venn diagram can be used to show the number of pupils that passed their Maths and Biology exams in Year 9.

There are 320 pupils in Year 9.

235 passed their Maths exam. 196 passed Biology. 103 passed both.



a) Complete the Venn diagram.

b) How many pupils passed neither exam?

c) How many students passed exactly one exam?

d) A student that passed Maths is chosen at random. What is the probability they also passed Biology?

Calculate the following:

i) P(M)

ii) P(B')

iii) P(M∩B')

Problem Solving

Tom's sock drawer contains 14 socks. 5 are black, 3 are white and the rest are grey.
He picks out 2 socks at random.
Is he more likely to pick out a pair that match or a pair that don't match?
You must show your working.

Delta Unit 13: Ratio and Proportion

Prior Knowledge

Simplify and find equivalent ratios.

Write and use unitary ratios.
E.G. Express 5:8 in the form 1:n
5:8 = 1:1.6

Share in a ratio.
E.G. Share £32 in the ratio 3:5
3+5=8 Parts
32÷8 =4. Each part is worth 4
4 x 3 =12 : 4 x 5 = 20
Answer = 12:20

Write a ratio as a fraction.
E.G. There are blue and green beads in a bag. They are in the ratio 4:7. What fraction of the beads are blue?
4+7 = 13
Therefore, fraction of blue = $\frac{4}{13}$.

Use ratios in the context of maps and scales.
E.G. The size of a model of a car to the life sized car is in the ratio 1:30. If the door on the car is 4cm wide, what is the width of the door on the life sized car?
4 x 30 = 120cm or 1.2m

Use ratio when scaling recipes.

Ratios for conversions

The key for using conversion ratios is knowing when to multiply or divide by the conversion.

The conversion rate for £ to € is 1:1.09. So every £1 is worth €1.09.

To convert £ to € we multiply. To convert € to £ we divide.

E.G. Convert £120 to euros   Answer: 120 x 1.09 = £130.80

E.G. Convert €330 to pounds   Answer: 330 ÷ 1.09 = £302.75

Top Tips:

- Does your answer make sense? Should it be bigger or smaller than the original amount.
- When asked to compare prices, you need to convert them to the same units and make them comparable.

Ratios and fractions

Be prepared to switch between ratios and fractions

E.G.
A factory makes small vans and big vans in the ratio 3:8.
The small vans are either blue or white in the ratio 2:5.
What fraction of the fines made are small and white?

The fraction which are small = $\frac{3}{11}$

The fraction of small vans which are white = $\frac{5}{7}$

The fraction which are white and small are $\frac{3}{11} \times \frac{5}{7} = \frac{15}{77}$

Literacy

Explain what is meant by equivalent ratios.

Reasoning

A question states: The ratio to red sweets to blue sweets in a bag is 3:4. What is the fraction of red sweets?

Chloe says. "The fraction of red sweets is $\frac{3}{4}$ "

Comment on Chloe's answer

.

Fluency

Divide in a ratio

1) Divide £144 in the ratio 7 : 5

2) Divide £42 in the ratio 4 : 3

Using map scales

1)Using a scale of 1 : 50 000 how many cm on a map would represent 0.5 km?

2) Using a scale of 1 : 50 000 how many km are represented by 11 cm on a map?

Using exchange rates

£1 = $1.50 USA £1 = €1.40 £1=$2.10 AUS

1) Change $84 AUS into GBP(£)

2) Change £400 into USA ($)

2) 3) Change $75 USA into GBP(£)

4) Change $42 AUS into USA ($)

Problem Solving

The ratio of green to black counters in a bag is 5 : 6 and the ratio of black to yellow counters is 3 : 4 . If there are 2 more black than green counters how many counters are there altogether?

Delta Unit 14: Direct and inverse proportion

Prior Knowledge

Use unitary proportion.

Understand scaling recipes.

Rearrange formulae.

Substitute into expressions and formulae.

Recognise direct proportion.

Recognise a graph in the form y=kx.

Find the constant of proportionality in direct and inverse proportion questions.

Solve equations.

Direct and Inverse Proportion involving squares and cubes

Numbers can be proportional to the square, cube, square root etc. of a number.

E.G. Y is directly proportion to X^2, when Y is 100, x is 5, find Y when X is 3.

Formula: $Y \propto X^2$ $(\propto \textit{means proportional to})$

$Y = kX^2$ (k is the constant of proportionality)

$100 = k5^2$

$4 = k$

So the formula linking y and x is $y = 4x^2$

To find Y substitute in $x = 5$ $\quad y = 4 \times 3^2 \quad y = 36$

E.G. Y is inversely proportion to $\sqrt[3]{x}$, when Y is 5, x is 8, find Y when X is 64

Formula: $Y \propto \frac{1}{\sqrt[3]{x}}$ $(\propto \textit{means proportional to})$

$Y = \frac{k}{\sqrt[3]{x}}$ (k is the constant of proportionality)

$5 = \frac{k}{\sqrt[3]{8}} \quad 10 = k$

So the formula linking y and x is $y = \frac{10}{\sqrt[3]{x}}$

To find Y substitute in $x = 5$ $\quad y = \frac{10}{\sqrt[3]{64}} \quad y = 2.5$

Exam Questions from worded scenarios

E.G.
A pebble is thrown vertically upwards.

It has an initial speed u metres per second.
The pebble reaches maximum height h metres before falling vertically downwards.
It is known that h is directly proportional to u^2.

When the pebble is thrown with an initial speed 10m/s it reaches a height of 5m.

Calculate the height it would reach when thrown at an initial speed 12m/s.

$h \propto u^2$ means $h = ku^2$

$u = 10m/s \quad h = 5m$

$5 = k10^2$

$5 = k \times 100$

$k = 0.05$

$h = 0.05u^2$

When $u = 12m/s$

$h = 0.05 \times 12^2$

$h = 7.2m$

Literacy

Explain what is meant by direct and inverse proportion.

Fluency

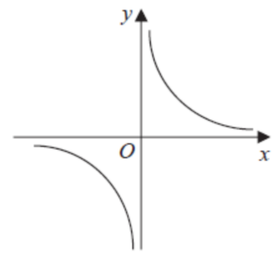
y is directly proportional to *x*,
y = 48 when x = 4
Find:
a)The equation for y and x
b) y when x = 8

y is directly proportional to x^3,
y = 216 when x = 3.
Find
a) The equation for y and x
b) X when y = 64

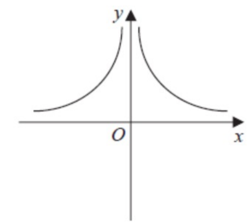
y is inversely proportional to *x*,
y = 5 when x = 8
Find:
a)The equation for y and x
b) y when x = 5

Problem Solving

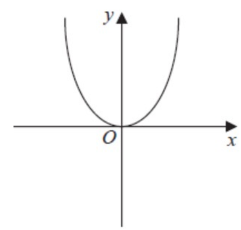
These graphs show four different proportionality relationships between *y* and *x*.
Match each graph with a statement in the table below.



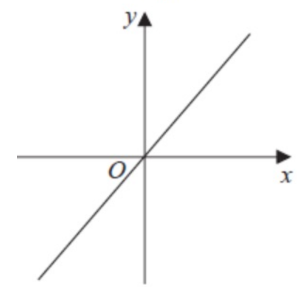
Graph A



Graph C



Graph B



Graph D

Proportionality relationship	Graph letter
y is directly proportional to *x*	
y is inversely proportional to *x*	
y is proportional to the square of *x*	
y is inversely proportional to the square of *x*	

Delta Unit 15: Polygons and Angles

Prior Knowledge

Know names of shapes both 2D and 3D.

Be able to use shape notation.

Basic angle facts:

- Angles on a straight line sum to 180°.
- Angles round a point sum to 360°.
- Angles in a triangle sum to 180°.
- Angles in a quadrilateral sum to 360°.
- Base angles in an isosceles triangle are equal.
- Vertically opposite angles are equal.

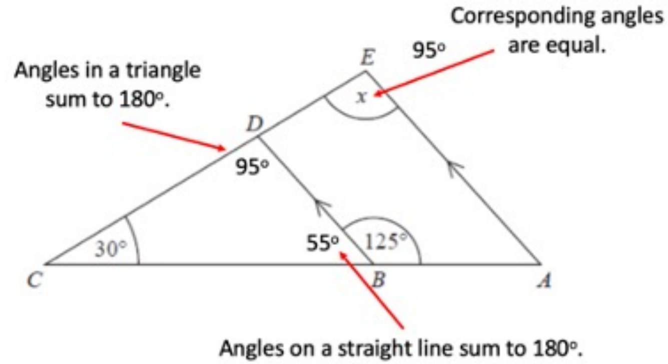
Angles in parallel lines

- Alternate angles are equal.
- Corresponding angles are equal.
- Co-interior angles sum to 180°.

Multi-step angle problems

You must show every step of working and give a full written reason for each angle you calculate.

E.G.



Angles and algebra

E.G.

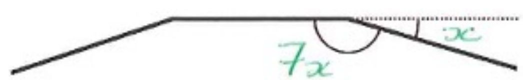


Diagram **NOT** accurately drawn

The diagram shows part of a regular polygon.
The interior angle and the exterior angle at a vertex are marked.
The size of the interior angle is 7 times the size of the exterior angle.

Work out the number of sides of the polygon.

Label the exterior angle x:

$$7x + x = 180\ (angles\ on\ straight\ line\ sum\ to\ 180^0)$$
$$8x = 180$$
$$x = 22.5$$

$$number\ of\ sides = \frac{360}{22.5} = 15\ sides \quad (exterior\ angles\ sum\ to\ 360^0)$$

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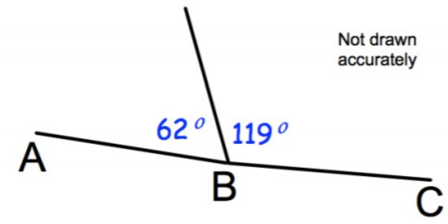
Literacy

Explain what is meant by faces, edges and vertices.

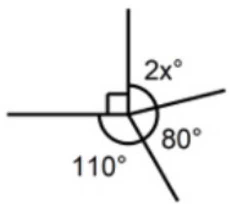
Reasoning

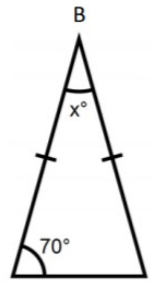
Bernard says AC is a straight line.
Is he correct? Explain your answer.

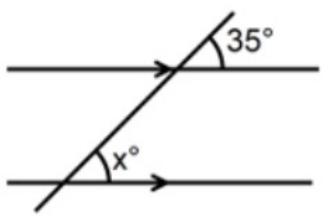


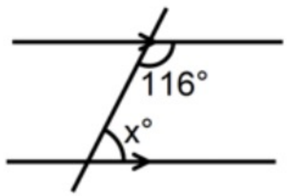
Fluency

Work out the size of the angle marked *x*.
Give reasons for your answer.



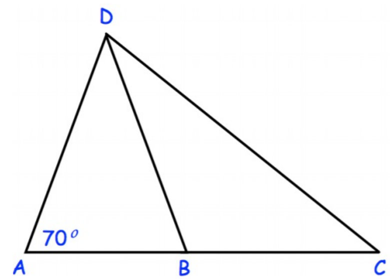






Problem Solving

Triangles ABD and BCD are both isosceles. AC is a straight line. Is ADC a right angle? Clearly explain your answer.



Delta Unit 16: Pythagoras and Trigonometry

Prior Knowledge

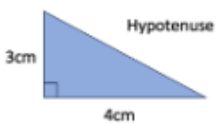
Square and square root using your calculator and without.

Solve equations.

Recognise right-angled triangles.

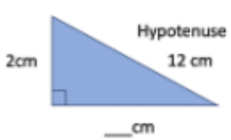
To be able to use Pythagoras' theorem.

To find the length of the hypotenuse you should square and add.



$$a^2 + b^2 = c^2$$
$$3^2 + 4^2 = c^2$$
$$25 = c^2$$
$$\sqrt{25} = c$$
$$5 = c$$

To find the length of a shorter side you should square and subtract.



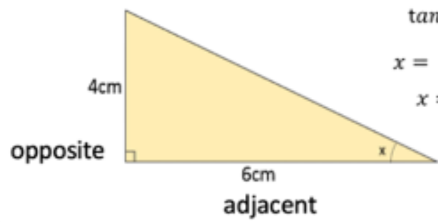
$$a^2 + b^2 = c^2$$
$$2^2 + b^2 = 12^2$$
$$4 + b^2 = 144$$
$$b^2 = 140$$
$$b = \sqrt{140}$$
$$b = 11.83$$

Trigonometry

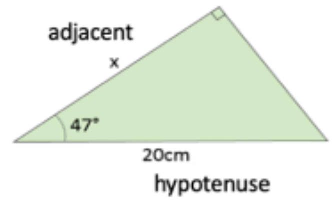
Trigonometry involves calculating angles and sides in triangles.
Trigonometry involves three ratios - sine, cosine and tangent which are abbreviated to sin, cos and tan.

$$sin\theta = \frac{opposite}{hypotenuse} \qquad cos\theta = \frac{adjacent}{hypotenuse} \qquad tan\theta = \frac{opposite}{adjacent}$$

E.G.



$$tan\theta = \frac{opposite}{adjacent}$$
$$\tan(x) = \frac{4}{6}$$
$$x = \tan^{-1}\left(\frac{4}{6}\right)$$
$$x = 33.7°$$



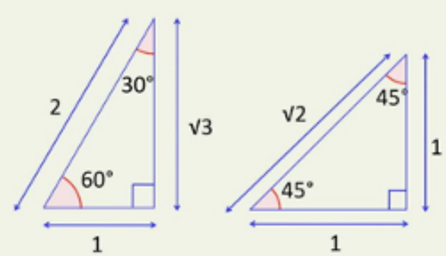
$$cos\theta = \frac{adjacent}{hypotenuse}$$
$$\cos(47) = \frac{x}{20}$$
$$20 \times \cos(47) = x$$
$$13.64cm = x$$

Exact values

You need to know these exact values and be able to work with them.

Angle (θ) Degrees	$0°$	$30°$	$45°$	$60°$	$90°$
$\sin(\theta)$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos(\theta)$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\tan(\theta)$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not Defined

You might find it easier to remember the triangles.



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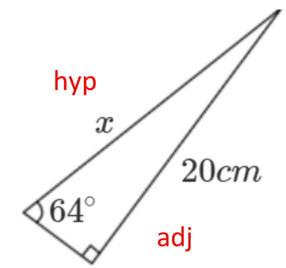
Literacy

Give the definition of hypotenuse.

Reasoning

Explain the mistakes that were made when calculating the solution to this question and find the correct solution.

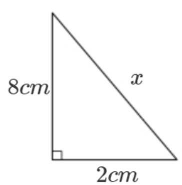
X = 20 x cos (64)
X = 8.8 cm



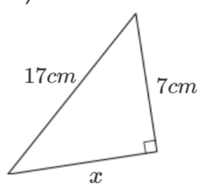
Fluency

Pythagoras

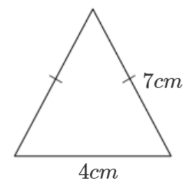
Find x $(2dp)$:
Q1)



Find x to the nearest cm:
Q1)

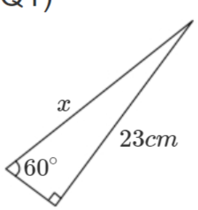


Find perimeter & area $(2dp)$:
Q1)

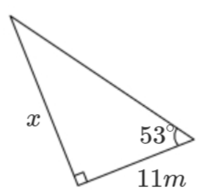


Trigonometry

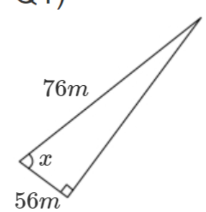
Find x $(2dp)$:
Q1)



Find x $(2dp)$:
Q1)



Find x $(2dp)$:
Q1)



Problem Solving

The diagram shows the positions of three turbines *A*, *B* and *C*.
A is 6 km due north of turbine *B*.
C is 4.5 km due west of turbine *B*.
(a) Calculate the distance *AC*.

(b) Calculate the bearing of *C* from *A*.
Give your answer correct to the nearest degree.

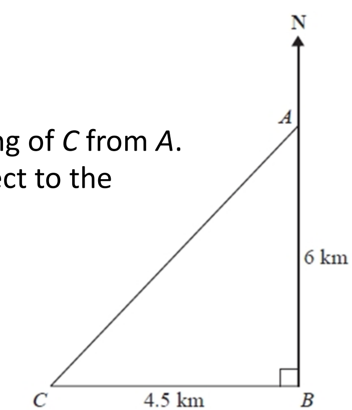


Diagram **NOT** accurately drawn