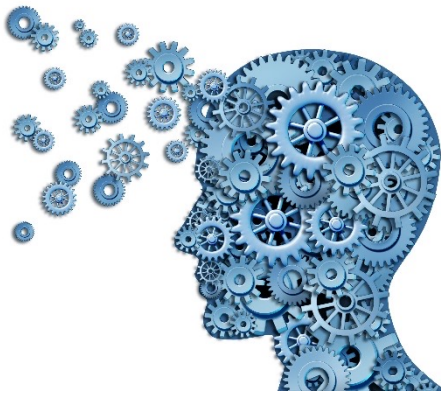


Kettlethorpe HIGH SCHOOL

MATHS Year 11 | Delta

Name:

Set:



| Unit | Topic | Complete |
|------|----------------------|----------|
| 1 | Collecting data | |
| 2 | Presenting data | |
| 3 | Circle theorems | |
| 4 | Circle geometry | |
| 5 | Changing the subject | |
| 6 | Vectors | |

Delta Unit 1: Collecting Data

Essential knowledge:

- Use sampling to estimate population sizes (capture-recapture method)
- Identify Bias

Key Words:

Sample, population, fraction, decimal, percentage, bias, stratified sample, random, box plot, histogram, frequency density, frequency, mean, median, mode, range, lower quartile, upper quartile, interquartile range, spread, comparison, outlier

Prior Knowledge:

- Equivalent fractions e.g.
$$\frac{5}{20} = \frac{30}{x}$$
$$x = \frac{30}{5} \times 20 = 120$$
- Know the difference between discrete and continuous data
- Know what bias is
- Know how to reduce bias in sampling and questionnaires
- Know that a larger sample produces more reliable results

Sampling (capture-recapture)

The capture-recapture method is used to estimate a **population** size from a **sample** size, by assuming that the sample size is in the same proportion as to the population size

Example

A man catches 20 fish in a lake, he marks them with a cross, he puts the fish back in the lake. The next day the man catches 30 fish and 5 of them have a cross on them. Estimate how many fish are in the lake.

Currently $\frac{5}{30}$ are marked, we know 20 are marked altogether.

So $\frac{5}{30} = \frac{20}{x}$ As they are in the same proportion, $x =$ the population size. To calculate $30 \times (20 \div 5) = 120$.

Example

Gary wants to have a party for his 300 friends, he asks 30 of them what their favourite crisps are and they give the following results

| Salt & Vinegar | Ready Salted | Cheese & Onion | Prawn Cocktail |
|----------------|--------------|----------------|----------------|
| 10 | 7 | 8 | 5 |

How many of each flavour should Gary order?

Gary sampled 30 out of 300, so the multiplier is $300 \div 30 = 10$. So for each flavour we multiply by 10

| Salt & Vinegar | Ready Salted | Cheese & Onion | Prawn Cocktail |
|----------------------|--------------------|--------------------|--------------------|
| $10 \times 10 = 100$ | $7 \times 10 = 70$ | $8 \times 10 = 80$ | $5 \times 10 = 50$ |

Caution

We have made assumptions for the above questions

We have assumed that the sample is representative of the population.

LITERACY

Write the definition of Random

Use the word random within a sentence

REASONING

Stephen traps 30 deer in the forest, tags them and releases them. A week later he traps 50 and 15 are tagged. He uses capture/recapture to estimate the total number of deer as 100. Write down 3 assumptions he must make.

FLUENCY

1) There are 477 people at a concert.

| | Male | Female |
|-------|------|--------|
| Adult | 57 | 83 |
| Child | 114 | 223 |

Eric wants to pick a sample of 80 stratified by gender and age.

- Work out the number of adult males in the sample.
- Work out the number of female children in the sample.

2) Taymar wants to estimate the number of fish in a lake. She catches 60 fish from the lake and marks them with a dye. She then releases the fish back in to the lake. The next day, Taymar catches 70 fish from the lake, 8 of the fish have been marked with the dye. Work out an estimate for the number of fish in the lake.

PROBLEM SOLVING

There are 2480 people in a town.

| | Men | Women | Children | Total |
|------------------|------|-------|----------|-------|
| Number in town | 1260 | | | 2480 |
| Number in sample | 63 | 22 | | |

The stratified sample is selected with Men, Women and Children being the 3 strata. Complete the table.

Delta Unit 2: Presenting data

Essential knowledge:

- Plot and interpret a cumulative frequency diagram
- Plot and interpret a Histogram

Key Words:

box plot, histogram, frequency density, frequency, mean, median, mode, range, lower quartile, upper quartile, interquartile range, spread, comparison, outlier, estimate

Prior Knowledge:

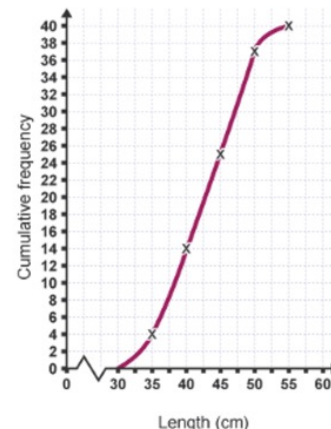
- Compare two distributions using an average and a measure of spread.
- Understand what the median is
- Understand what upper and lower quartiles are
- Calculate inter-quartile range
- Understand that with grouped data we are estimating the distributions within the groups
- Calculate mean from grouped data
- Identify outliers

Constructing a Cumulative Frequency Diagram

Cumulative frequency creates a running total of the amounts within a table, it is used for continuous grouped data. A cumulative frequency diagram is created by plotting the upper class boundary with the cumulative frequency.

| Length (cm) | Frequency | Cumulative frequency |
|------------------|-----------|----------------------|
| $30 < l \leq 35$ | 4 | 4 |
| $35 < l \leq 40$ | 10 | $14 (4 + 10 = 14)$ |
| $40 < l \leq 45$ | 11 | $25 (14 + 11 = 25)$ |
| $45 < l \leq 50$ | 12 | $37 (25 + 12 = 37)$ |
| $50 < l \leq 55$ | 3 | $40 (37 + 3 = 40)$ |

Remember you need to plot 0 at the lower class boundary for the first group. In addition, the graph should be plotted with straight lines.



Reading from Cumulative Frequency Diagram

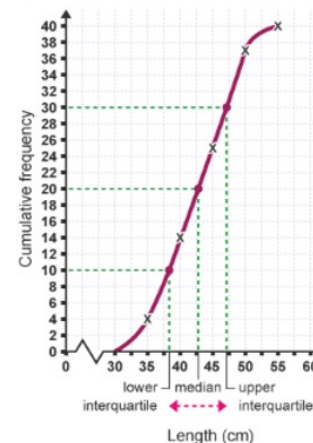
We can estimate the median and quartiles from a cumulative frequency diagram,

Median – Draw a line across from the middle value e.g. the above data has 40 people so the median will be the 20th value, if we read across from there we have a median of 43

Upper/Lower Quartile - Draw a line across for the marker for top and bottom 25%. E.g., the lower quartile is the tenth person giving us 38, upper quartile would be the 30th person, giving us 47

Interquartile range – this is the upper quartile take the lower quartile e.g. $47 - 38 = 9$

Remember: Values taken from cumulative frequencies are estimates, as we do not know the distribution of the data within the groups.



LITERACY

Write the definition of Outlier

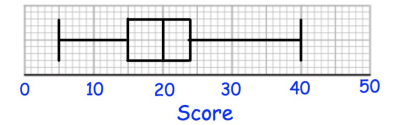
Use the word outlier within a sentence

REASONING

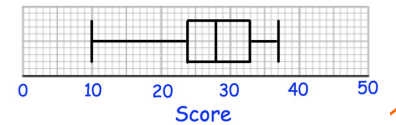
Compare the two classes results.

(a)

7A results



7B results

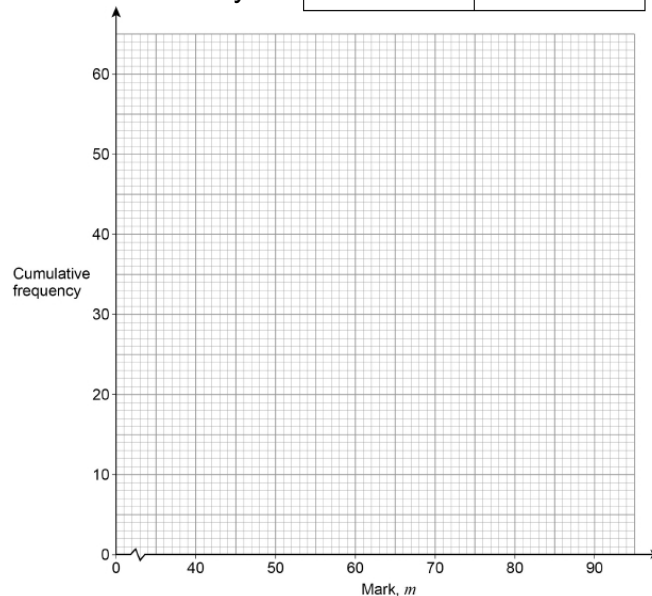


FLUENCY

Here is some information about students marks on a test.

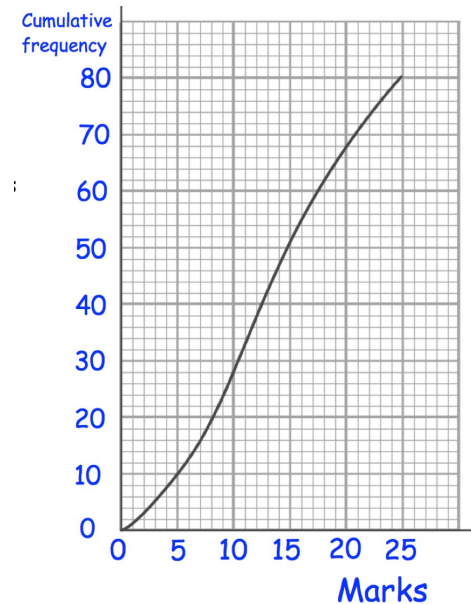
- Draw a cumulative frequency diagram for this information.
- Estimate the median mark achieved by a pupil.
- Calculate the IQR.

| Mark, m | Frequency |
|------------------|-----------|
| $40 < m \leq 50$ | 9 |
| $50 < m \leq 60$ | 16 |
| $60 < m \leq 70$ | 20 |
| $70 < m \leq 80$ | 8 |
| $80 < m \leq 90$ | 7 |



PROBLEM SOLVING

What percentage of people scored more than 80% of the marks?



Delta Unit 3: Circle Theorems

Essential knowledge:

- Know and apply the circle theorems

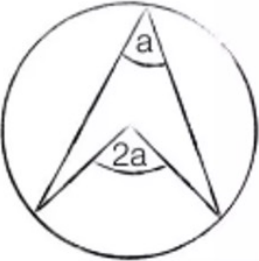
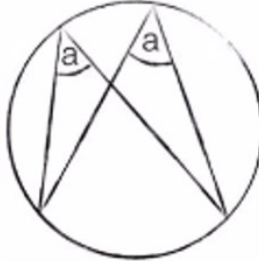
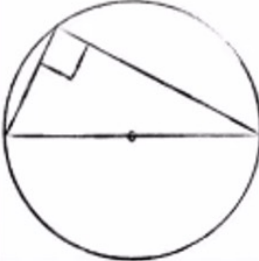
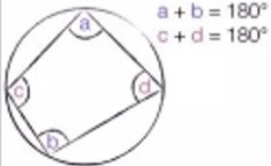
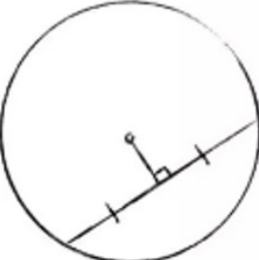

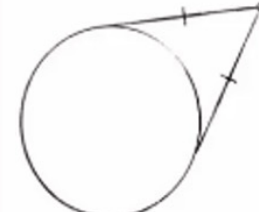
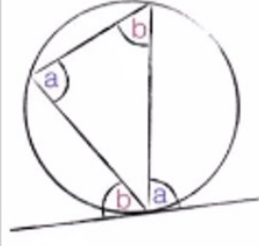
Key Words:

Radius, centre, tangent, circumference, diameter, gradient, perpendicular, reciprocal, coordinate, equation, substitution, chord, triangle, isosceles, angles, degrees, cyclic quadrilateral, alternate, segment, semicircle, arc, theorem

Prior Knowledge:

- Now the basic angle laws e.g. Angles on a straight line
- Construct your reasoning for a missing angle problem
- Identify the names for parts of a circle
- Construct a proof

Circle Theorems

| | | | |
|---|--|---|--|
| <p>The angle at the centre is twice the angle at the circumference</p>  | <p>Angles in the same segment are equal</p>  | <p>The angle in a semicircle is 90 degrees</p>  | <p>Opposite angles in a cyclic quadrilateral add up to 180 degrees</p>  |
| <p>The perpendicular from the centre to the chord bisects the chord</p>  | <p>The angle between a tangent and a radius is 90 degrees</p>  | <p>Tangents from a point outside a circle are equal in length</p>  | <p>Alternate segment theorem</p>  |

Example 1

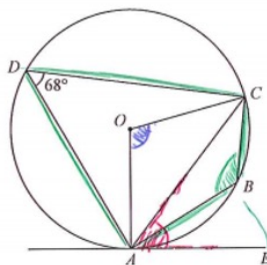


Diagram NOT accurately drawn

A, B, C and D are points on a circle, centre O.
AE is a tangent to the circle.
Angle $ADC = 68^\circ$

- (a) (i) Find the size of angle ABC.

$180 - 68$

112° (AI)

- (ii) Give a reason for your answer.

OPPOSITE ANGLES IN A CYCLIC QUADRILATERAL
ADD TO 180° (2) (AI)

- (b) (i) Find the size of angle AOC.

2×68

136° (AI)

- (ii) Give a reason for your answer.

ANGLE AT THE CENTRE IS TWICE ANGLE AT THE CIRCUMFERENCE (2) (AI)

- (c) Find the size of angle CAE.

(ANGLE BETWEEN A TANGENT AND CHORD IS EQUAL TO THE ANGLE IN THE ALTERNATE SEGMENT)

68° (AI)

Example 2

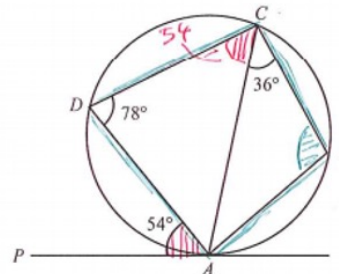


Diagram NOT accurately drawn

A, B, C and D are points on a circle.
PA is the tangent to the circle at A.
Angle $PAD = 54^\circ$, angle $ACB = 36^\circ$ and angle $ADC = 78^\circ$.

- (a) (i) Find the size of angle ACD.

54° (AI)

- (ii) Give a reason for your answer.

ANGLES IN ALTERNATE SEGMENTS ARE EQUAL (2) (AI)

- (b) Explain why BD is a diameter of the circle.

BECAUSE $\hat{BCD} = 36 + 54 = 90^\circ$ AND ANGLES FROM DIAMETERS ARE 90° (2) (AI)

- (c) (i) Work out the size of angle ABC.

$180 - 78$

102° (AI)

- (ii) Give a reason for your answer.

OPPOSITE ANGLES IN A CYCLIC QUADRILATERAL ADD UP TO 180° (2) (AI)

LITERACY

Explain the following words:

Estimated mean-

Modal class interval-

REASONING

A football team played six games.

Here are the number of goals they scored in each game:

6 0 3 2 2 5

The football team play one more game.

The mean number of goals scored increases to 4.

(c) Work out the number of goals scored in the seventh game.

FLUENCY

4. Timothy asked 30 people how long it takes them to get to school.



The table shows some information about his results.

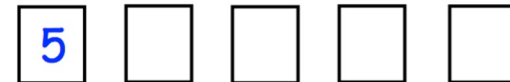
| Time (t minutes) | Frequency |
|------------------|-----------|
| $0 < t \leq 10$ | 2 |
| $10 < t \leq 20$ | 8 |
| $20 < t \leq 30$ | 12 |
| $30 < t \leq 40$ | 7 |
| $40 < t \leq 50$ | 1 |

Work out an estimate for the mean time taken.

.....minutes

PROBLEM SOLVING

Shown below are five cards which are arranged in order from smallest to largest



The range of the cards is 4.
The median of the cards is 8.
The mean of the cards is 7.

Work out the 4 missing numbers.

Delta Unit 4: Circle Geometry

Essential knowledge:

- Find the equation of a tangent to a circle at a given point.

Key Words:

Radius, centre, tangent, circumference, diameter, gradient, perpendicular, reciprocal, coordinate, equation,

Prior Knowledge:

- To be able to draw a circle using a compass
- To know the general form for the equation of a circle

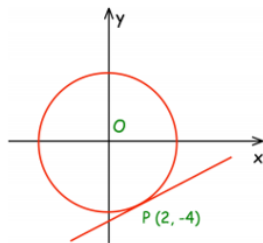
$$x^2 + y^2 = r^2$$

- Plot the graph of a circle
- Find the gradient of a line given two points
- Find the gradient of a perpendicular line using the negative reciprocal
- Find the equation of a line given a point and gradient

Equation of a tangent

Example:

Here is a circle, centre O, and the tangent to the circle at the point (2, -4).



Find the equation of the tangent at the point P.

First: Find the gradient of the radius between O and P: $\frac{-4-0}{2-0} = -\frac{4}{2} = -2$

Second: Find the gradient of the line perpendicular to this: $\frac{1}{2}$

Third: Find the equation of the tangent using the new gradient and the point P

$$y = mx + c \quad -4 = -2\left(\frac{1}{2}\right) + c \quad -4 = -1 + c \quad -3 = c$$

$$y = \frac{1}{2}x - 3 \quad \text{another form would be } 2y = x - 6$$

Hint: Be careful you may be asked to leave your answer in a certain form e.g. $ay + bx = c$

LITERACY

Explain the following words:

Estimated mean-

Modal class interval-

REASONING

A football team played six games.

Here are the number of goals they scored in each game:

6 0 3 2 2 5

The football team play one more game.

The mean number of goals scored increases to 4.

(c) Work out the number of goals scored in the seventh game.

FLUENCY

4. Timothy asked 30 people how long it takes them to get to school.



The table shows some information about his results.

| Time (t minutes) | Frequency |
|------------------|-----------|
| $0 < t \leq 10$ | 2 |
| $10 < t \leq 20$ | 8 |
| $20 < t \leq 30$ | 12 |
| $30 < t \leq 40$ | 7 |
| $40 < t \leq 50$ | 1 |

Work out an estimate for the mean time taken.

.....minutes

PROBLEM SOLVING

Shown below are five cards which are arranged in order from smallest to largest



The range of the cards is 4.
The median of the cards is 8.
The mean of the cards is 7.

Work out the 4 missing numbers.

Delta Unit 5: Changing the subject

Essential knowledge:

- Rationalise the denominator with surds
- Manipulate algebraic fractions
- Find inverse functions and combined functions
- Use algebraic proof

Key Words:

Rationalise, denominator, surd, rational, irrational, fraction, equation, rearrange, subject, proof, function notation, inverse, evaluate, numerator, denominator, input, output

Prior Knowledge:

- Expand and factorise single and double brackets
- Add and subtract fractions
- Multiply and divide fractions
- Find equivalent fractions
- Simplify expressions involving surds
- Substitute into expressions and formulae
- Know about the difference of 2 squares
- Basic function notation
- Substitute an expression into another expression

Rationalise the denominator

Rationalise the denominator is to remove the surd element from the denominator, it is done by multiplying by the surd.

$$\begin{aligned} \text{e.g. (i)} \quad \frac{4}{\sqrt{2}} &= \frac{4}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} \\ &= \frac{4\sqrt{2}}{2} \\ &= 2\sqrt{2} \end{aligned}$$

For some you use difference of 2 squares

$$\begin{aligned} \text{E.g.} \quad \frac{6}{3+\sqrt{7}} &= \frac{6}{3+\sqrt{7}} \times \frac{3-\sqrt{7}}{3-\sqrt{7}} \\ \frac{18-6\sqrt{7}}{9-7} &= \frac{18-6\sqrt{7}}{2} = 9 - 3\sqrt{7} \end{aligned}$$

Algebraic Fractions

The basic rules of fractions apply when algebra is involved, e.g. to add fractions they need the same denominator

$$\begin{aligned} \frac{3p}{p-5} - \frac{4}{2p+1} &= \frac{3p \times (2p+1)}{(p-5) \times (2p+1)} - \frac{4 \times (p-5)}{(p-5) \times (2p+1)} \\ &= \frac{6p^2 + 3p}{(p-5) \times (2p+1)} - \frac{4p - 20}{(p-5) \times (2p+1)} \\ &= \frac{6p^2 - p + 20}{2p^2 - 9p - 5} \end{aligned}$$

Hint: The way to a common denominator is by multiplying each fraction by the opposing fractions denominator.

Function Notation

Inverse Functions are the reverse function to find the function that would find the input. Example: $F(X) = 2x + 3$, Find $F'(X)$

$$\begin{aligned} Y &= 2x + 3 \\ Y - 3 &= 2x \\ \frac{Y-3}{2} = x &\Rightarrow F'(x) = \frac{x-3}{2} \end{aligned}$$

Compound function, this is where you combine two functions, by substituting one function into the other. Example: $f(x) = 2x + 3$, $g(x) = 5x - 6$, find $gf(x)$

$$\begin{aligned} gf(x) &= 5(2x+3) - 6 = 10x + 15 - 6 \\ gf(x) &= 10x + 9 \end{aligned}$$

Algebraic Proof

You must know and be able to apply these algebraic representations

Number = n ,
Consecutive numbers = $n, n+1$, etc.
Even number = $2n$,
Odd number = $2n+1$,

To prove an expression is a multiple of n , then you must be able to factorise an n out of the expression

E.g. $12n+9$ is a multiple of 3 because

$12n+9 = 3(4n+3)$ and anything multiplied by 3 is a multiple of 3.

LITERACY

Explain the following words:

Estimated mean-

Modal class interval-

REASONING

A football team played six games.

Here are the number of goals they scored in each game:

6 0 3 2 2 5

The football team play one more game.

The mean number of goals scored increases to 4.

(c) Work out the number of goals scored in the seventh game.

FLUENCY

4. Timothy asked 30 people how long it takes them to get to school.



The table shows some information about his results.

| Time (t minutes) | Frequency |
|------------------|-----------|
| $0 < t \leq 10$ | 2 |
| $10 < t \leq 20$ | 8 |
| $20 < t \leq 30$ | 12 |
| $30 < t \leq 40$ | 7 |
| $40 < t \leq 50$ | 1 |

Work out an estimate for the mean time taken.

.....minutes

PROBLEM SOLVING

Shown below are five cards which are arranged in order from smallest to largest



The range of the cards is 4.
The median of the cards is 8.
The mean of the cards is 7.

Work out the 4 missing numbers.

Delta Unit 6: Vectors

Essential knowledge:

- Understand vector notation for geometric problems.
- Construct a geometrical proof using vectors

Key Words:

Vector, direction, magnitude, scalar, multiple, parallel, collinear, proof, ratio, column vector

Prior Knowledge:

- Understand column vectors
- Add and subtract column vectors
- Convert between ratios and fractions
- Manipulate fractions
- Construct a proof
- Find the length of a line using Pythagoras theorem
- Know the properties of triangles and quadrilaterals
- Manipulate algebraic expressions

Vector Notation

Vectors have both direction and magnitude. You combine two vectors (adding them)

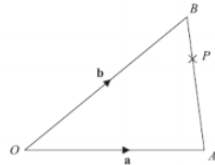


Diagram NOT accurately drawn

OAB is a triangle.

$$\begin{aligned}\vec{OA} &= \mathbf{a} \\ \vec{OB} &= \mathbf{b}\end{aligned}$$

(a) Find \vec{AB} in terms of \mathbf{a} and \mathbf{b} .

To go from A to B, we go A to O ($-\mathbf{a}$) then O to B
(b)

$$\vec{AB} = \mathbf{b} - \mathbf{a}$$

P is the point on AB such that $AP : PB = 3 : 1$

(b) Find \vec{OP} in terms of \mathbf{a} and \mathbf{b} .
Give your answer in its simplest form.

\vec{AP} is $\frac{3}{4}$ the distance of AB , therefore

$$\vec{AP} = \frac{3}{4}(\mathbf{b} - \mathbf{a})$$

Vector Proofs

To prove that 2 vectors are parallel you need to show that one is a multiple of another

Example:

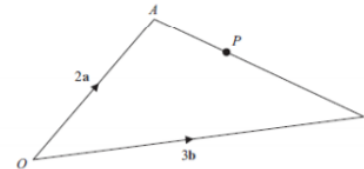


Diagram NOT accurately drawn

OAB is a triangle.

$$\vec{OA} = 2\mathbf{a}$$

$$\vec{OB} = 3\mathbf{b}$$

P is the point on AB such that $AP : PB = 2 : 3$

(b) Show that \vec{OP} is parallel to the vector $\mathbf{a} + \mathbf{b}$.

$$\begin{aligned}\vec{AB} &= 3\mathbf{b} - 2\mathbf{a}, \quad \vec{AP} = \frac{2}{5}(3\mathbf{b} - 2\mathbf{a}) \\ \vec{OP} &= \vec{OA} + \vec{AP} = 2\mathbf{a} + \frac{2}{5}(3\mathbf{b} - 2\mathbf{a}) = 2\mathbf{a} + \frac{6}{5}\mathbf{b} - \frac{4}{5}\mathbf{a}\end{aligned}$$

$$\vec{OP} = \frac{6}{5}\mathbf{a} + \frac{6}{5}\mathbf{b} = \frac{6}{5}(\mathbf{a} + \mathbf{b})$$

Therefore, OP is parallel to $\mathbf{a} + \mathbf{b}$

To prove that 3 points are in a straight line, e.g. MNC you show that MN is a multiple of MC , therefore parallel and using a common point

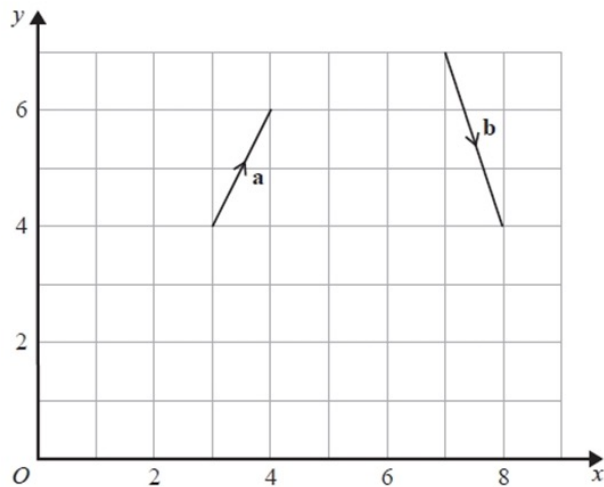
LITERACY

Write the definition of collinear.

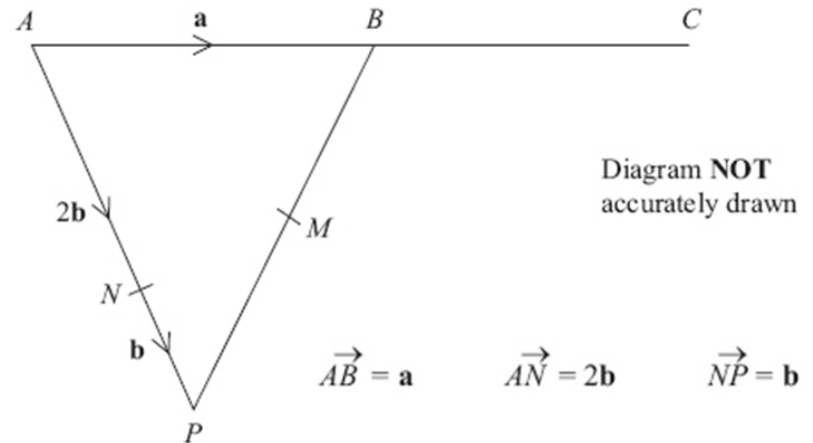
FLUENCY

The vector \mathbf{a} and the vector \mathbf{b} are shown on the grid.

- On the grid, draw and label vector $-2\mathbf{a}$
- Work out $\mathbf{a} + 2\mathbf{b}$ as a column vector.



PROBLEM SOLVING



APB is a triangle

N is a point on AP

- Find the vector \vec{PB} in terms of \mathbf{a} and \mathbf{b} .

B is the midpoint of AC

M is the midpoint of PB

- Show that NMC is a straight line.