

Kettlethorpe

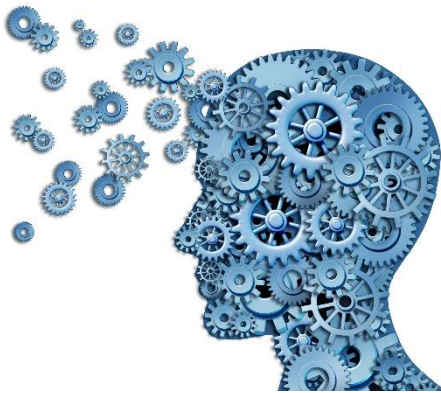
HIGH SCHOOL

MATHS

Year 10 | Theta

Name:

Set:



Unit	Topic	Complete
1	Percentages	
2	Compound Measures	
3	Averages	
4	Tables, Charts and Graphs	
5	Pie Charts and Scatter Graphs	
6	Simultaneous Equations	
7	Sequences	
8	Linear Graphs	
9	Quadratic Graphs	
10	Inequalities	
11	Area and Perimeter	
12	Trigonometry	
13	3D Shapes	
14	Real Life Graphs	
15	Constructions	

Theta Unit 1: Percentages

Prior Knowledge

Convert between fractions decimals and percentages.

Express a given number as a percentage of another number, including where the percentage is greater than 100%.

E.G.

Express $\frac{57}{60}$ as a percentage $\frac{57}{60} = \frac{19}{20} = \frac{95}{100} = 95\%$.

Calculate percentage change with or without a multiplier.

E.G.

Increase 70 by 30%

Without multiplier, find $30\% = 21$ add this to original $70 + 21 = 91$.

With a multiplier $100\% + 30\% = 130\% = 1.3$
 $70 \times 1.3 = 91$.

Find an original quantity after a percentage change.

E.G.

The cost of a ticket has been increased by 12.5% to £225, find the original amount.
 $£225 \div 1.125 = £200$.

Compound Percentages

Example

James invests £6000 for 5 years, at 3% a year.
At the end of 5 years, calculate the value of the investment after 5 years.

Answer:

$$100\% + 3\% = 103\% = 1.03$$

$$6000 \times 1.03^5$$

$$£6955.64$$

Example 2

Katy invests £2000 for 3 years.
She receives interest per year of 2.5% in the first year then $x\%$ for the second and third year.
There is a total of £2124.46 at the end of the 3 years. Work out the value of x .

Answer:

Let y be the multiplier for the second and third year.

$$2000 \times 1.025 \times y^2 = 2124.46$$

$$y = \sqrt{\frac{2124.46}{2000 \times 1.025}}$$

$$y = 1.017999$$

Therefore the increase, x , is 1.8%.

Literacy

Write the definition of percentage.

Write a question about finding a percentage of an amount that you could be asked to answer.

Reasoning

There are 800 students at Prestfield School. 144 of these were absent on Wednesday.

- How many attended on Wednesday?
- Trudy says more than 25% were absent, is this true?

Fluency

Without a calculator.

- 10% of £80 =
- 10% of £130 =
- 10% of £459 =

With a calculator.

- 28% of £200 =
- 63% of £500 =
- 19% of £400 =

Jack invests \$400 in an account paying 3% interest for 5 years.

How much does he have in the account after 5 years?

Problem Solving

The table gives information about an estate agents charges when selling houses.

Value of the house	Estate agent's charges
Up to £60 000	2% of the value of the house
Over £60 000	2% of the first £60 000 plus 1% of the remaining value of the house

The agent sold a home for £80 000.
Work out the charge.

Theta Unit 2: Compound Measures

Prior Knowledge

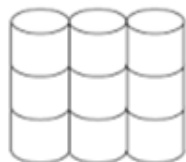
- Use simple unitary proportion.
- Understand scaling recipes.
- Rearrange formulae.
- Solve equations.
- Work out best buys.

Best Buys

E.G.



Pack of 4
toilet rolls
£1.96



Pack of 9
toilet rolls
£4.23

Which pack offers the best value for money?

We need to work out the cost for 1 toilet roll from each.

Option 1 (9 rolls)
1 toilet roll costs
 $£4.23 \div 9 = 47p$.

Option 2 (4 rolls)
1 toilet roll costs
 $£1.96 \div 4 = 49p$.

Conclusion:
Option 1 is better value.

You must always show your working on these questions.

Simple Compound measures

You must know and be able to use the formulas and be able to rearrange them when needed.

$$\text{speed} = \frac{\text{distance}}{\text{time}} \quad \text{density} = \frac{\text{mass}}{\text{volume}}$$

$$\text{pressure} = \frac{\text{force}}{\text{area}}$$

You will also need to be able to convert between their measures, i.e. convert between mph and miles per minute.

Harder Compound measures

Use a table to help you organise your working out. This is helpful when there are multiple parts to journeys.

Example:

Sienna travels from Birmingham to Leeds at an average speed of 60mph.
She then travels from Leeds to Darlington at an average speed of 40mph.
The distance from Birmingham to Leeds is 150miles.
The distance from Leeds to Darlington is 70 miles.
Calculate Sienna's average speed.

	B TO L	L TO D	TOTAL B TO D
SPEED	60mph	40mph	51.76mph
DISTANCE	150miles	70miles	220miles
TIME	2.5h	1.75h	4.25h

Literacy

Write the definition of density.

Write the definition of mass.

Reasoning

If wood has density less than 1g/cm^3 it will float.
Which of these will be best for building a toy boat?

Plank A

Volume = 750cm^3
Mass = 900g

Plank B

Volume = 0.0152m^3
Mass = 7.6kg

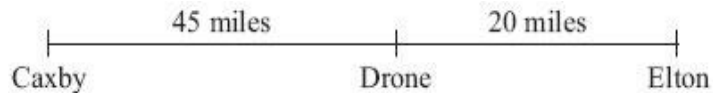
Plank C

Volume = 1000cm^3
Mass = 1.02kg

Fluency

The distance from Caxby to Drone is 45 miles.

The distance from Drone to Elton is 20 miles.



Colin drives from Caxby to Drone. Then he drives from Drone to Elton.

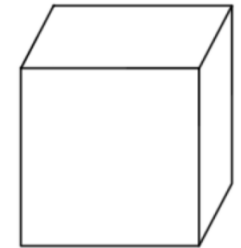
Colin drives from Caxby to Drone at an average speed of 30 mph.

He drives from Drone to Elton at an average speed of 40 mph.

Work out Colin's average speed for the whole journey from Caxby to Elton.

Problem Solving

The diagram below shows a solid block of ice.



81cm

A block of ice weighs $\frac{1}{2}$ tonne.

The block is a cube with side length 81cm.

Find the density of the ice.

Give your answer in kilograms per cubic metre.

Theta Unit 3: Averages

Prior Knowledge

Understand the difference between quantitative and qualitative data.

Understand the difference between continuous and discrete data.

Put data into a frequency table.

Put data into a grouped frequency table.

Find the averages, mode, mean and median for a data set.

Find the range for a data set.

Compare data using an average and range.

Median and Mode from frequency table

Here is a table showing the number of goals scored in 10 football matches.

Number of goals	Frequency
0	2
1	2
2	5
3	1

Mode = 2 (the class with highest frequency)

The **median** is the class containing the 5.5th data point.

Number of goals	Frequency	Cumulative
0	2	2
1	2	2+2 = 4
2	5	4 + 5 = 9
3	1	9 + 1 = 10

The 5.5th data point is in the category for 2, therefore the median is 2.

Mean from Frequency Table

To find the mean, you need to find the total number of goals scored.

Number of goals, g	Frequency, f	g x f
0	2	0
1	2	2
2	5	10
3	1	3

Total goals $0 + 2 + 10 + 3 = 15$.

Mean = $15/10 = 1.5$ goals per game.

Remember

When the data is grouped like below, we estimate the mean using the midpoint for the classes.

Length (l, cm)	Midpoint (M)	Frequency, f	M x f
$10 < l \leq 20$	15	10	150
$20 < l \leq 40$	30	30	900
$40 < l \leq 50$	45	20	900

Estimated total length = $150 + 900 + 900 = 1950$.

Estimated Mean = $1950 \div 60 = 32.5$ goals per game.

Literacy

Explain the meaning of quantitative and qualitative data.

Reasoning

Decide if the statements are true or false. Give a reason for each of your answers.

Score	Frequency
11	3
12	3
13	2
14	3
15	1

A The data set contains exactly 10 values.

B The median is 13 because it's in the middle of: 11, 12, 13, 14, 15.

C The mean can't be 12.7 because only whole numbers could be scored.

D The mean score is 2.4.

E The data is discrete.

F The modal value is 3.

Fluency

Calculate the mean, median and mode from each table.

(a)

Age	Frequency
5	2
6	2
7	5
8	1

(b)

Number of phones	Frequency
0	1
1	3
2	2
3	0
4	4
5	0

(c)

Mass	Frequency
$20 < m \leq 25$	12
$25 < m \leq 30$	24
$30 < m \leq 35$	17
$35 < m \leq 40$	15
$40 < m \leq 45$	4

(d)

Height	Frequency
$120 < h \leq 130$	51
$130 < h \leq 140$	120
$140 < h \leq 150$	66
$150 < h \leq 160$	59
$160 < h \leq 170$	4

Problem Solving

Test Scores		
	BOYS	GIRLS
Mean	32 marks	40 marks
Range	18	15

Compare the distributions of boys and girls test scores.

Theta Unit 4: Representing Data

Prior Knowledge

Understand the difference between quantitative and qualitative data.

Understand the difference between continuous and discrete data.

Compare data using an average and range.

Draw and interpret bar charts.

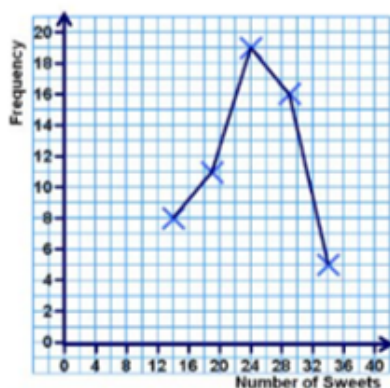
Draw and interpret composite and dual bar charts.

Frequency Polygons

- Plot midpoints.
- Join dot-to-dot.

E.G.

Number of Sweets	Mid Value	Frequency
12 - 16	14	8
17 - 21	19	11
22 - 26	24	19
27 - 31	29	16
32 - 36	34	5



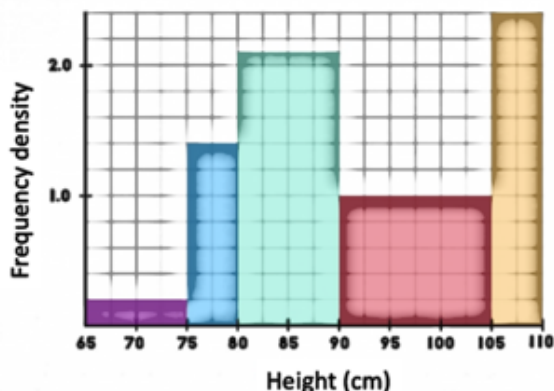
Constructing a Histogram

Histograms are used to represent continuous data. We plot bars that are the width of the classes, but for the height, we use frequency density, which is given by the formula:

$$\text{Frequency density} = \frac{\text{frequency}}{\text{class width}}$$

Example

HEIGHT (CM)	FREQUENCY	CLASS WIDTH	FREQUENCY DENSITY
$65 < h \leq 75$	2	10	$2/10 = 0.2$
$75 < h \leq 80$	7	5	$7/5 = 1.4$
$80 < h \leq 90$	21	10	$21/10 = 2.1$
$90 < h \leq 105$	15	15	$15/15 = 1$
$105 < h \leq 110$	12	5	$12/5 = 2.4$



Literacy

Explain the meaning of discrete and continuous data.

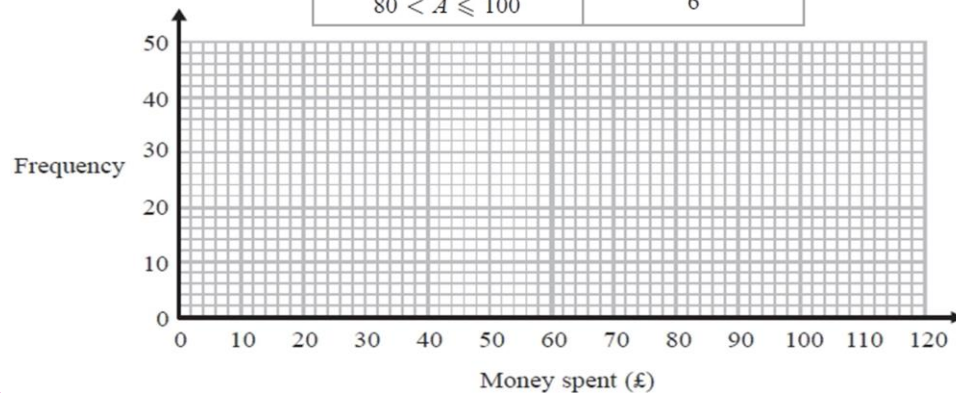
Reasoning

Dahlia says “if a bar on a histogram is twice as tall as another bar, then it represents twice as many people”
Is she correct? Give a reason for your answer.

Fluency

The table gives information about the money, £ A , some people spent on an internet site one day. On the grid, draw a frequency polygon for this information.

Money spent (£ A)	Frequency
$0 < A \leq 20$	10
$20 < A \leq 40$	15
$40 < A \leq 60$	25
$60 < A \leq 80$	40
$80 < A \leq 100$	6



Problem Solving

Theta Unit 5: Pie Charts and Scatter Graphs

Prior Knowledge

Construct angles.

Draw a pie chart.

E.g.

The table shows the pets 40 people own:

$$1 \text{ person} = 360^\circ \div 40 = 9^\circ$$

Type of pet	Frequency	Angle
Dog	17	$17 \times 9 = 153^\circ$
Cat	12	$12 \times 9 = 108^\circ$
Rabbit	5	$5 \times 9 = 45^\circ$
Other	6	$6 \times 9 = 54^\circ$

$$360^\circ \div 40 = 9$$

Draw your pie chart with the correct angles and label each section:

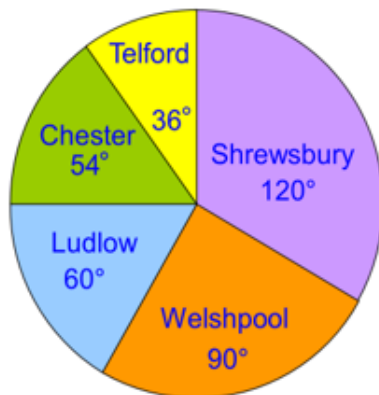


Interpret Pie Charts

When interpreting pie charts, always work out the degrees per person first.

E.G.

The following pie chart shows the amount of people travelling to certain destinations.



15 people are going to Welshpool, how many are going to Telford?

$$90 \div 15 = 6^\circ$$

$$36 \div 6 = \underline{6 \text{ people}}$$

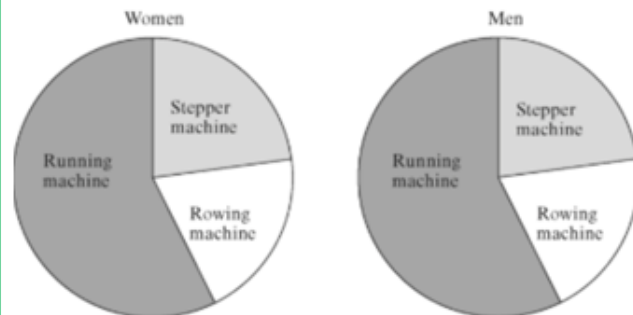
Misleading Pie Charts

Remember pie charts can be misleading as they represent proportions and if you don't know the total, they can be misleading.

E.G.

Men and women at a gym were asked which of these fitness machines they spent the most time on.

The gym instructor produced these pie charts.



Tomas states, "Men and women spend the same time on each machine".

Give a reason why Tomas may be wrong.

Although the proportions are the same, we don't know if there are the same amount of men and women.

Literacy

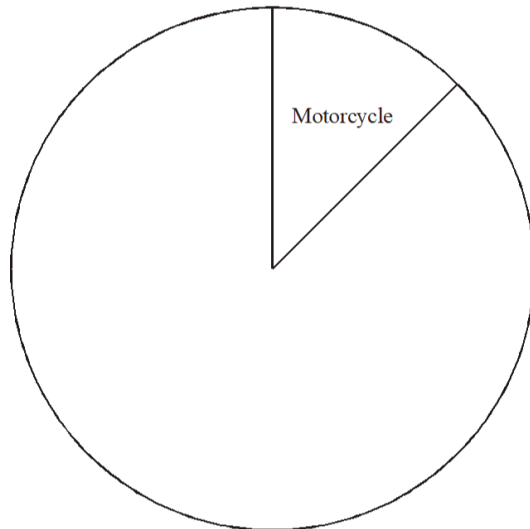
Explain the difference between a population and a sample.

Fluency

The table shows information about 40 vehicles crossing a bridge.

Type of vehicle	Number of vehicles	Size of angle
Motorcycle	5	45°
Car	16	
Bus	11	
Other	8	

Complete the pie chart to show this information.

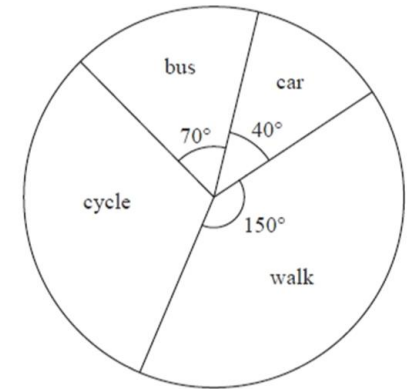


Reasoning

The pie chart shows information about how the students in Year 11 get to school.

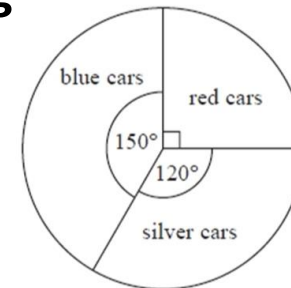
Mr Morley says "Less than 10% of Year 11 get to school by car".

Is Mr Morley correct?

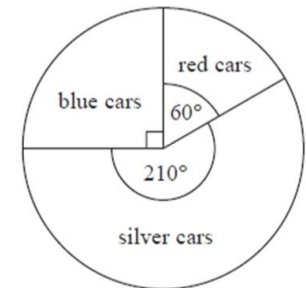


Problem Solving

Allerton School



Bragdon School



There are only silver cars, blue cars and red cars in the car parks at Allerton School and at Bragdon School.

The pie charts show information about the numbers of these cars.

- (a) What fraction of the cars in the car park at Allerton School are blue?
Give your fraction in its simplest form.

.....

There are 12 red cars in the car park at Bragdon School.

- (b) How many silver cars are there in this car park?

Theta Unit 5: Pie Charts and Scatter Graphs

Prior Knowledge

Plot coordinates.

Read diagrams.

Plot a scatter graph.

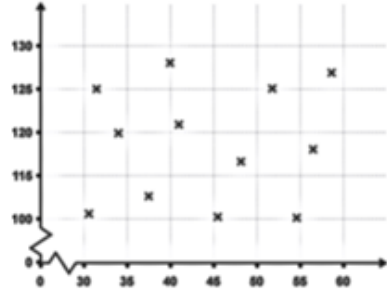
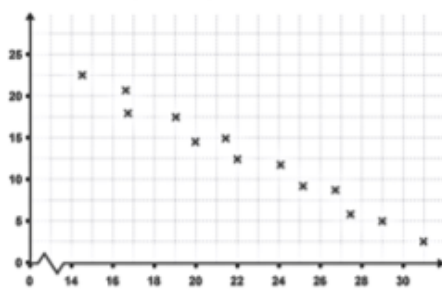
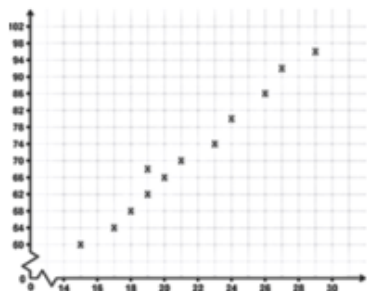
Interpreting Correlation

A frequency polygon is used to plot grouped data, it is plotted as mid-point against frequency, the points are then joined using a ruler.

Positive correlation means as one variable increases so does the other variable.

Negative correlation means as one variable increases the other variable decreases.

No correlation means there is no relationship between the two variables.



Interpolation and Extrapolation

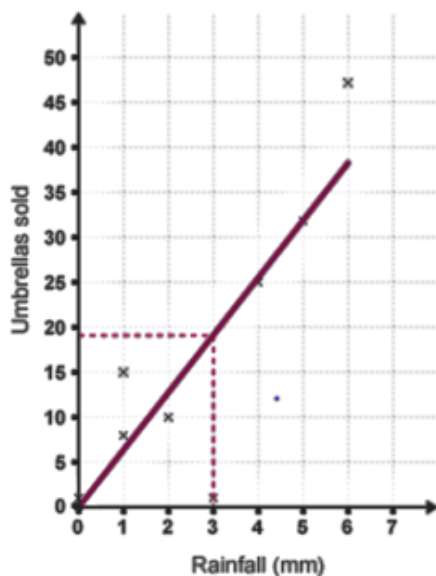
Scatter diagrams can be used to make estimates, first we need to draw a line of best fit.

Example:

For the data in the example, estimate how many umbrellas would be sold when there is 3mm of rain.

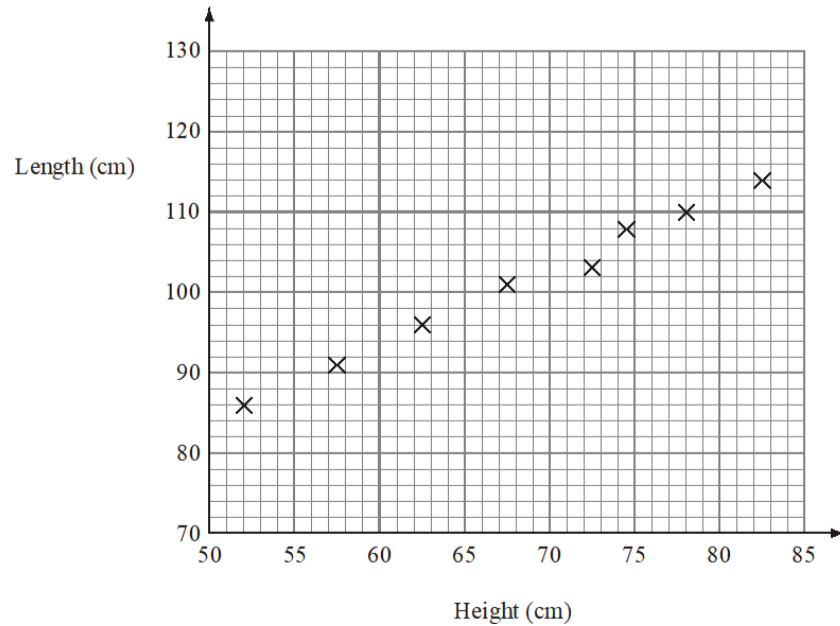
Therefore reading from our line of best fit, we'd estimate 19 umbrellas to be sold. This is **interpolation** as it is within the data range we have.

When we are asked to do this outside of the data range it is called **Extrapolation** and is less accurate as we have no data in this range.



Fluency

The scatter graph shows information about eight sheep. It shows the height and the length of each sheep.



The table gives the height and the length of two more sheep.

Height (cm)	65	80
Length (cm)	100	110

- (a) On the scatter graph, plot the information from the table.
- (b) Describe the relationship between the height and the length of these sheep.

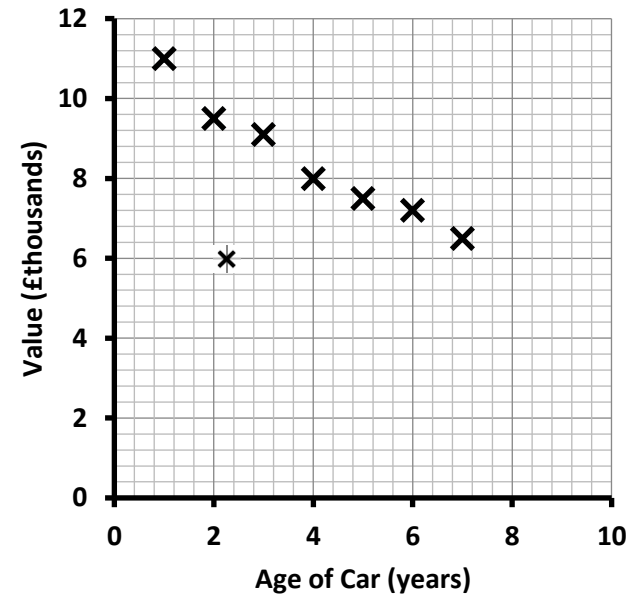
.....

Literacy

Give an example of two variables that you might expect to have positive correlation.

What does the term outlier mean?

Reasoning



One of these points is an outlier. Which one and why?

Theta Unit 6: Solving Simultaneous Equations

Prior Knowledge

- Substitute into expressions.
- Solve linear equations.
- Expand double brackets.
- Manipulate expressions.
- Factorise quadratics.
- Square rooting gives 2 solutions.
- Roots are solutions to quadratics.

Solving quadratic equations using the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Linear Simultaneous Equations

Simultaneous equations involve 2 variables, and you need a solution for each variable, you eliminate one variable first.

E.G.

Solve the simultaneous equations:

$$3x + 4y = 19$$

$$5x + 6y = 30$$

$$3x + 4y = 19 \quad \times 3 \rightarrow \quad 9x + 12y = 57$$

$$5x + 6y = 30 \quad \times 2 \rightarrow \quad 10x + 12y = 60$$

$$-1x + 0y = -3$$

$$x = 3$$

Substitute $x = 3$ into one equation to find the value of y :

$$3(3) + 4y = 19$$

$$9 + 4y = 19$$

$$4y = 10$$

$$y = 2.5$$

Check $y = 2.5$ $x = 3$ in the other equation.

$5 \times 3 + 6 \times 2.5 = 15 + 15 = 30$ so is correct.

So our solution is $y = 2.5$ and $x = 3$

Hint: When solving graphically, plot both lines and see where they intersect.

Non-Linear Simultaneous Equations

Simultaneous equations involve 2 variables, and you need a solution for each variable, you first need to substitute.

E.G.

Solve the simultaneous equations:

$$y = 2x + 4$$

$$x^2 + y^2 = 40$$

Substitute $y = 2x + 4$ into one equation to find the value of y :

$$x^2 + (2x + 4)^2 = 40$$

$$x^2 + 4x^2 + 16x + 16 = 40$$

$$5x^2 + 16x + 16 = 40$$

$$5x^2 + 16x - 24 = 0$$

Use the quadratic formula to solve to find x .

$$x = 1.11 \text{ or } x = -4.31$$

Substitute x to find y values.

$$y = 6.22 \text{ or } y = -4.62$$

Literacy

What is a coefficient?

Reasoning

How many points of intersection does the curve $y = (x - 3)(x + 4)$ have with the line $y = x - 8$?

Fluency

(a) $2x + 2y = 14$
 $5x - 3y = 19$

(b) $2x + 3y = 1$
 $7x + 2y = -22$

(c) $5x + 3y = 22$
 $2x + 4y = 20$

Problem Solving

The cost of buying a coffee and a tea in a cafe is £4. The cost of buying a coffee and three teas in a cafe is £7. Work out the cost of buying a coffee and the cost of buying a tea.

Theta Unit 7: Sequences

Prior Knowledge

Substitute numbers into expressions.

Substitute numbers into quadratic expressions.

Recognise simple sequences.

Work out the terms of an arithmetic sequence using the term-to-term rule.

Work out a given term in a simple arithmetic sequence.

Work out and use the nth term for an arithmetic sequence.

Generating a Quadratic Sequence

E.G.

Generate the first 4 terms of the sequence $2n^2 - 5$.

$2 \times 1^2 - 5 = -3$, $2 \times 2^2 - 5 = 3$, $2 \times 3^2 - 5 = 13$, $2 \times 4^2 - 5 = 27$ so the answer is $-3, 3, 13, 27$.

Nth Term of a linear sequence

The nth term is the rule that tells us how to find any term in the sequence.

E.G.

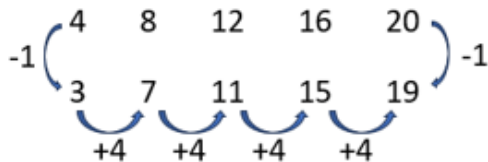
Find the nth term for 3, 7, 11, 15, 19, ...

Step 1: Sequence goes up by 4 so the nth term starts $4n$.

Step 2: Write out 4, 8, 12, 16, 20, ... above the sequence.

Step 3: The sequence is 1 less than the 4 times table so the nth term ends -1 .

This sequence is $4n - 1$



Finding the common ratio for a Geometric Sequence

A geometric sequence is not generated by adding an amount but by multiplying by the same value each time. The number which you multiply by each time is known as the common ratio.

To find the common ratio you divide two subsequent terms in a geometric sequence.

E.G.

1) 8, 24, 72, 216 in this geometric sequence the common ratio = $\frac{24}{8} = 3$

Literacy

Explain what a geometric sequence is?

What does the term 'generate' mean?

Reasoning

A sequence of numbers is shown below.

1 5 9 13 17

- (a) Find an expression for the n th term of the sequence.
(b) Explain why 95 will not be a term in this sequence.

Fluency

Find the n th term of these sequences:

- a) 6, 11, 16, 21
b) 7, 12, 17, 22
c) 4, 6, 8, 10
d) 4, 8, 12, 16
e) 5, 9, 13, 17
f) 10, 7, 4, 1

Generate the first 5 terms of these sequences

- a) $6n + 1$
b) $4n - 6$

Problem Solving

Here are the n th terms of 4 sequences.

Sequence 1	n th term	$3n + 1$
Sequence 2	n th term	$5n + 10$
Sequence 3	n th term	$10n$
Sequence 4	n th term	$5n - 1$

For each sequence state whether the numbers in the sequence are

- A Always multiples of 5
S Sometimes multiples of 5
N Never multiples of 5

Sequence 1

Sequence 2

Sequence 3

Sequence 4

Theta Unit 8: Linear Graphs

Prior Knowledge

Plot Linear graphs.

Identify the gradient, m , of a linear graph.

Find the equation of a straight-line graph in the form $y=mx + c$.

Identify parallel lines, they have the same gradient.

Identify perpendicular lines, the gradients are the negative reciprocal of one another.

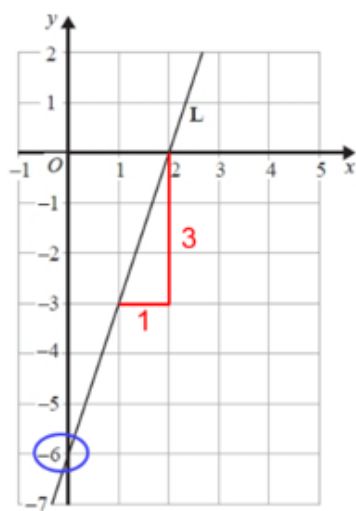
Equation of a Straight Line

The equation of a line is in the form:
 $y = mx + c$

M is the gradient. C is the y intercept

E.G.
 $m = 3 \div 1 = 3$
 $c = -6$ (crosses y -axis)

So $y = 3x - 6$

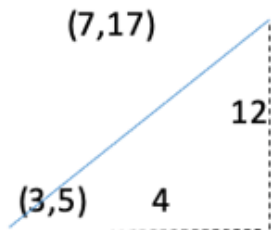


Equation of a Line Given Two Points

To find the equation of a line between two points, first it's good to draw a picture, find the gradient between the two points, then use this in the equation for m and use one point to find the intercept.

Example

Find the equation of the line that passes through the points $(3,5)$ and $(7, 19)$



$$\text{Gradient} = \frac{17-5}{7-3} = \frac{12}{4} = 3 \quad \text{so } y = 3x + c$$

$$5 = 3 \times 3 + c$$

$$5 = 9 + c$$

$$\text{Answer } y = 3x - 4$$

Literacy

Describe the term 'linear graph'.

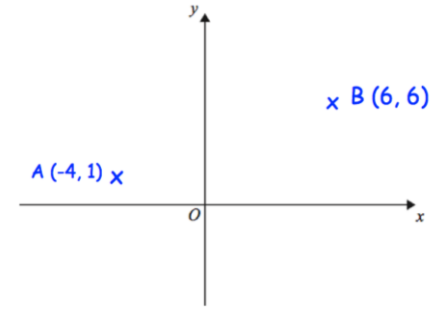
Describe the term 'substitute'.

Problem Solving

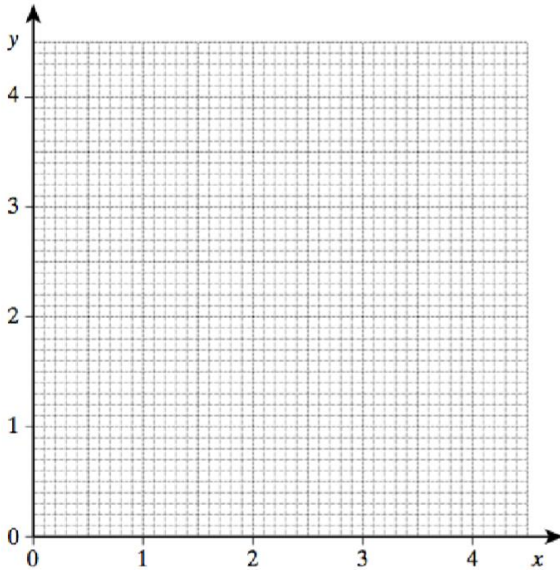
A is the point $(-4, 1)$

B is the point $(6, 6)$

Find the gradient of AB.



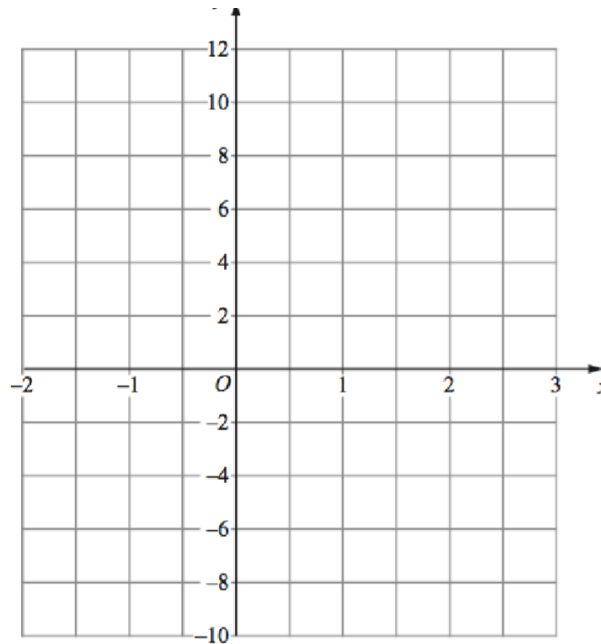
Fluency



(a) Complete the table of values for $y = 2x + 4$.

x	-1	0	1	2	3
y		4			10

(b) On the grid, draw the graph of $y = 2x + 4$ for values of x from -1 to 3 .

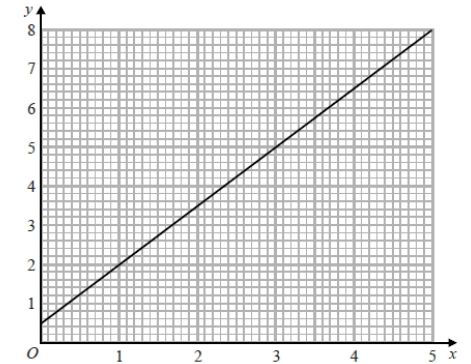


(a) On the grid, draw the graph of $x = 3$.

(b) On the grid, draw the graph of $y = 1$.

(c) Write down the coordinates of where the two lines met.

Reasoning



Phone calls cost $\pounds y$ for x minutes.

The graph gives the values of $\pounds y$ between 0 and 5 minutes.

Give an interpretation of the y intercept.

Give an interpretation of the gradient.

Theta Unit 9: Non-Linear Graphs

Prior Knowledge

Draw linear graphs.

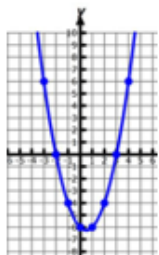
Solve linear equations.

Substitute into quadratic expressions.

Complete a table of values for and plot a quadratic graph.

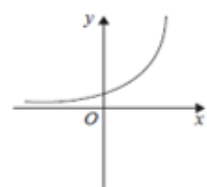
E.G. Plot the graph of $y = x^2 - x - 6$

x	y
-3	6
-2	0
-1	-4
0	-6
1	-6
2	-4
3	0
4	6

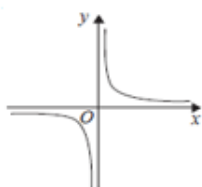


Key Graphs

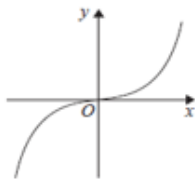
There are key graphs that you need to be able to recognise and know the general form.



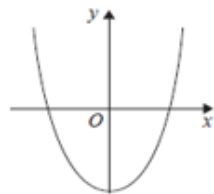
$$y = a^x$$



$$y = \frac{a}{x}$$



$$y = x^3$$



$$y = x^2$$

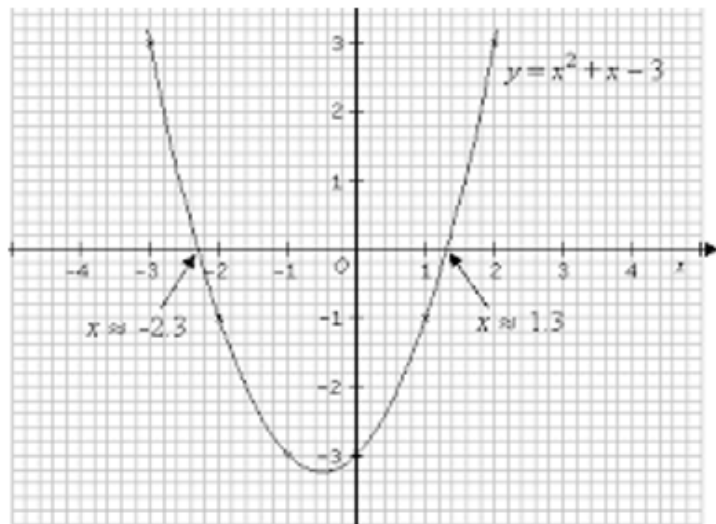
Solutions from Quadratic Graphs

This is where the graph crosses the x axis. They can be called solutions or roots.

Example

Given the graph $y = x^2 + x - 3$.

Roots: $x = -2.3$ or $x = 1.3$.



Literacy

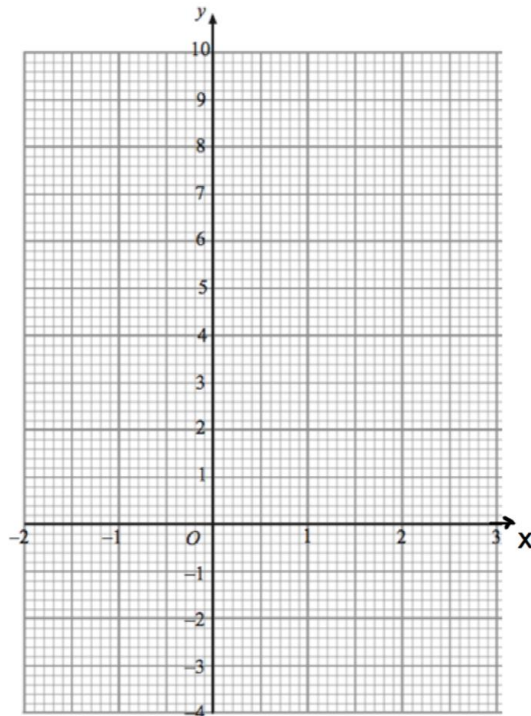
Explain the difference between a linear and a quadratic graph.

Fluency

Complete the table of values for $y = x^2 - 1$.

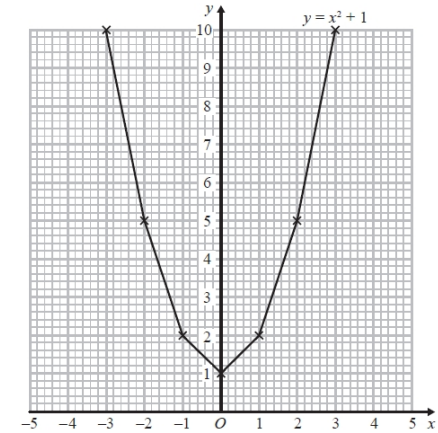
x	-2	-1	0	1	2	3
y	3		-1		3	

Draw the graph for $y = x^2 - 1$ on the graph below.



Reasoning

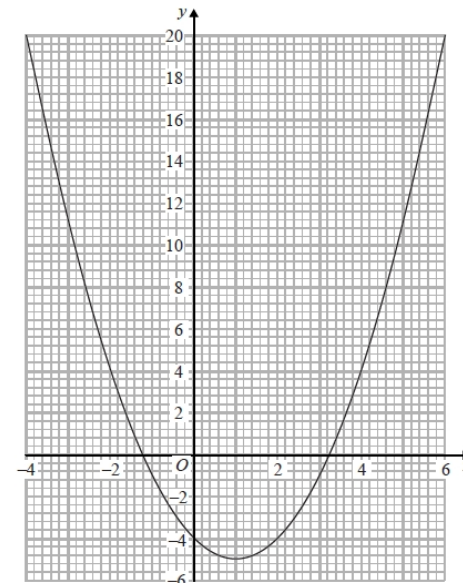
State one thing wrong with the quadratic graph.



Problem Solving

Here is the graph of $y = x^2 - 2x - 4$

(a) Write down estimates for the roots of $x^2 - 2x - 4 = 0$



Theta Unit 10: Inequalities

Prior Knowledge

Know what integer means (whole number).

Use inequality notation:

- $<$ means less than
- \leq means less than or equal to
- $>$ Means more than
- \geq means more than or equal to

Solve linear equations.

Substitute into expressions.

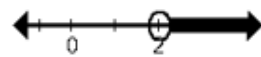
Factorise expressions.

Representing Inequalities on a Number Line

We can represent inequalities on a number line, hollow means not included, filled in means included.



$$x < 2, \text{ } x \text{ is less than } 2$$



$$x > 2 \text{ } x \text{ is greater than } 2$$



$$x \leq 2 \text{ } x \text{ is less than or equal to } 2$$



$$x \geq 2 \text{ } x \text{ is greater than or equal to } 2$$

Solving Inequalities

Sometimes we solve linear inequalities, which are solved the same as equations using balancing method.

E.G.

$$\text{Solve } 5x - 24 > 11$$

$$\begin{array}{r} 5x - 24 > 11 \\ +24 \quad +24 \\ \hline 5x > 35 \\ \div 7 \quad \div 7 \\ \hline x > 7 \end{array}$$

Solving inequalities gives a range of answers, rather than an individual solution.

Solving Two Linear Inequalities

Sometimes we have a group of two inequalities, you still use the balancing method, doing the same inverse operation to all 3 parts of the inequality.

E.G.

$$\text{Give the integers } 3 < 2x - 5 \leq 11$$

$$\begin{array}{r} 2 < 2x - 5 \leq 11 \\ +5 \quad +5 \quad +5 \\ \hline 7 < 2x \leq 16 \\ \div 2 \quad \div 2 \quad \div 2 \\ \hline 3.5 < x \leq 8 \end{array}$$

The integers (whole numbers) which satisfy this inequality are:

$$4, 5, 6, 7, 8.$$

Literacy

Write the definition of Inequality

Use the word inequality within a sentence

Reasoning

Solve $7 - 4x > 15$

Ama gives the following solution

$$11 - 8x > 15 \quad (-7)$$

$$8x > 4 \quad (\div 8)$$

$$x > 2$$

Comment on Ama's solution.

Fluency

List the integer solutions for the following:

1) $-2 < x \leq 5$ 2) $4 \geq x > -4$ 3) $-3 \leq 2x \leq 10$

Solve the following:

1) $2x + 7 \leq 15$ 2) $4x + 13 \geq 6$ 3) $5x - 2 \geq 2x + 13$

4) $2(x + 7) \leq 5(2x + 3)$ 5) $7 - 3x < 19$ 6) $12 - 2x > 8 - 5x$

Problem Solving

1) Given that a and b are integers such that

$$10 < 2a < 22$$

$$-3 < b < 6$$

and $a + b = 9$

Find all the possible values of a .

2) Bianca, Bob and Valentina have completed some Hegarty tasks. Bianca has completed 40 more than Bob. Valentina has completed 3 times as many as Bob. Together Bob and Bianca have completed twice as many as Valentina. Calculate the least number of tasks Bob could have completed.

Theta Unit 11: Area and Perimeter

Prior Knowledge

Perimeter is the distance around the outside of a 2D shape.

Area is the amount of space inside a 2D shape.

Identify the names for parts of a circle.

Calculate the area of basic and compound shapes.

Convert between metric measures for area.

Know the following key formulae:

Area triangle = $\frac{1}{2} \times \text{base} \times \text{perpendicular height}$

Area parallelogram = $\text{base} \times \text{perpendicular height}$

Area trapezium = $\frac{1}{2}(a + b) \times \text{height}$

Area of a circle = πr^2

Circumference of circle = πd

Area of Sector

The area of a sector is calculated by the formula:

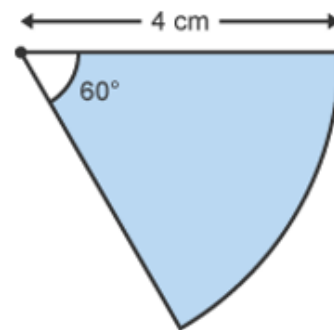
$$\frac{\theta}{360} \times \pi r^2,$$

where θ = the angle inside the sector.

Example

Find the area of the sector enclosed by two radii of 4cm and 60° .

$$\text{Area} = \frac{60}{360} \times \pi \times 4^2 = 8.4\text{cm}^2$$



Arc Length

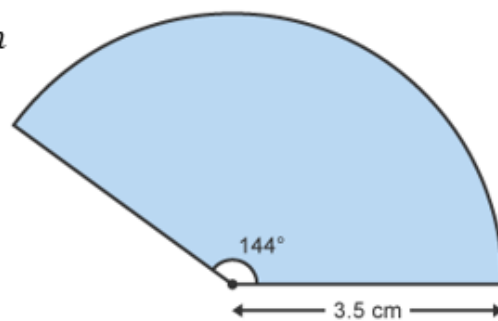
The arc length is calculated by the formula $\frac{\theta}{360} \times \pi 2r$ or $\frac{\theta}{360} \times \pi d$,

where θ = the angle inside the sector.

Example

Find the minor arc length, enclosed by radii of 3.5cm and 144° .

$$\text{Arc length} = \frac{144}{360} \times \pi \times 2 \times 3.5 = 8.8\text{cm}$$



Literacy

Write the definition of a Trapezium.

Use the word trapezium within a sentence.

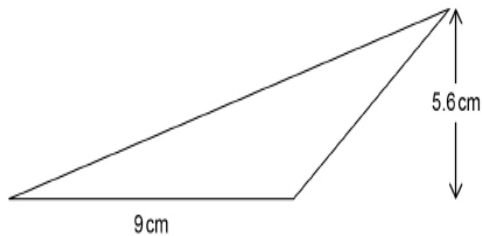
Reasoning

Sameera says that you cannot draw a square that has a perimeter of x cm and an area of x cm² (where x is the same value in each case). Is Sameera correct? Give a reason for your answer. .

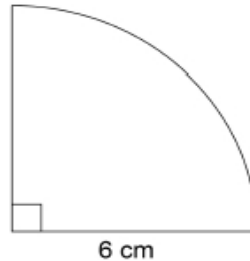
Fluency

Find the area of the following shapes:.

1)



2)



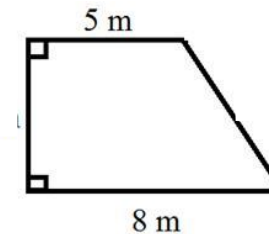
Find the area of a sector with radius 8cm and angle between the two radii of 82° .

Find the arc length of a sector with radius 4.6cm and an angle between the two radii of 145° .

Problem Solving

1) A circular pie is cut in to 8 slices of equal shape. The area of the top of one of these slices is 48cm². Find the diameter of the pie.

2) The area of this right-angled trapezium is 26m. Find the perimeter of the trapezium.



Theta Unit 12: Trigonometry

Prior Knowledge:

Square and square root using your calculator and without.

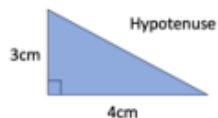
Solve equations.

Recognise right-angled triangles.

Be comfortable with the functions on your calculator.

To be able to use Pythagoras' theorem.

To find the length of the hypotenuse you should square and add.



$$a^2 + b^2 = c^2$$

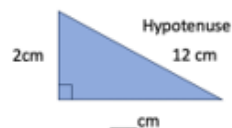
$$3^2 + 4^2 = c^2$$

$$25 = c^2$$

$$\sqrt{25} = c$$

$$5 = c$$

To find the length of a shorter side you should square and subtract.



$$a^2 + b^2 = c^2$$

$$2^2 + b^2 = 12^2$$

$$4 + b^2 = 144$$

$$b^2 = 140$$

$$b = \sqrt{140}$$

$$b = 11.83$$

Trigonometry

Trigonometry involves calculating angles and sides in triangles.

Trigonometry involves three ratios - sine, cosine and tangent which are abbreviated to sin, cos and tan.

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} \quad \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} \quad \tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

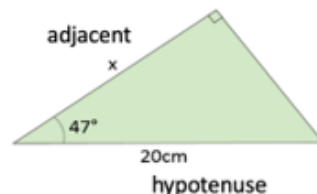
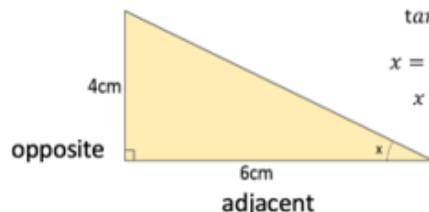
E.G.

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan(x) = \frac{4}{6}$$

$$x = \tan^{-1}\left(\frac{4}{6}\right)$$

$$x = 33.7^\circ$$



$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\cos(47) = \frac{x}{20}$$

$$20 \times \cos(47) = x$$

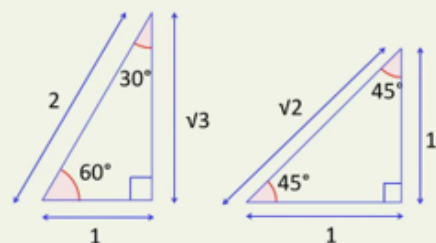
$$13.64\text{cm} = x$$

Exact values

You need to know these exact values and be able to work with them.

Angle (θ) Degrees	0°	30°	45°	60°	90°
$\sin(\theta)$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos(\theta)$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
$\tan(\theta)$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not Defined

You might find it easier to remember the triangles.



Literacy

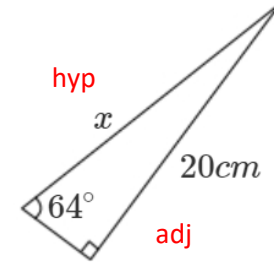
Give the definition of hypotenuse.

Reasoning

Explain the mistakes that were made when calculating the solution to this question and find the correct solution.

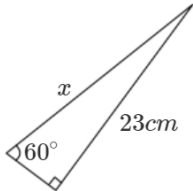
$$X = 20 \times \cos(64)$$

$$X = 8.8 \text{ cm}$$

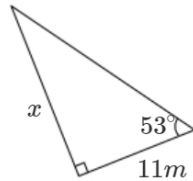


Fluency

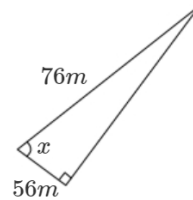
Find x (2dp):
Q1)



Find x (2dp):
Q1)



Find x (2dp):
Q1)



Problem Solving

The diagram shows the positions of three turbines A , B and C .

A is 6 km due north of turbine B .

C is 4.5 km due west of turbine B .

(a) Calculate the distance AC .

(b) Calculate the bearing of C from A .

Give your answer correct to the nearest degree.

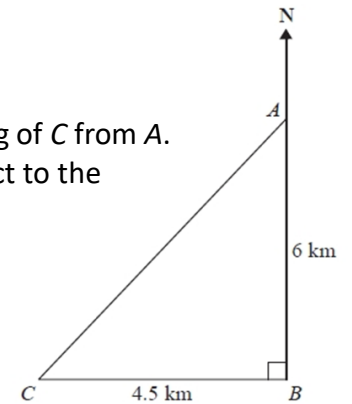


Diagram **NOT** accurately drawn

Theta Unit 13: 3D Shapes

Prior Knowledge

Volume is the space inside a 3D shape.

Calculate the volume of a prism, including cylinders.

Draw plans and elevations for shapes.

Convert between metric measurements for volume.

Calculate surface area of 3D shapes.

Spheres

If required to calculate the volume or surface area of a sphere then the following formulae will be given.

$$\text{Volume of sphere} = \frac{4}{3} \times \pi \times r^3$$

$$\text{Surface area of sphere} = 4 \times \pi \times r^2$$

Volume of Pyramid

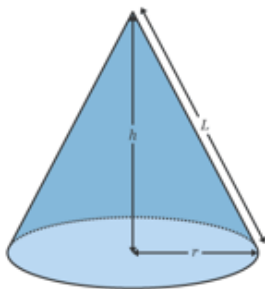
The formula volume of a pyramid will not be given to you. The volume is calculated from the formula.

$$\frac{1}{3} \times \text{area of base} \times \text{height}$$

Remember: a cone is a circular based pyramid

Curved Surface Area of Cones

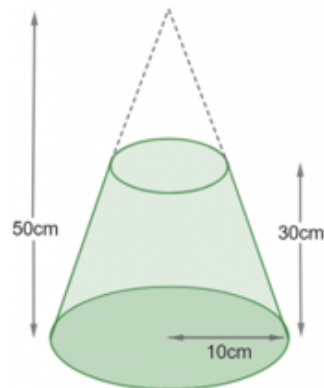
For the curved surface area of a cone the formula is $\pi \times r \times l$



Remember: for total surface area add the area of the base on.

Frustums

A frustum is a truncated pyramid, to calculate the volume, take the volume of the smaller cone away from the volume of the larger cone.



Literacy

Write the definition of prism.

Use the word prism within a sentence.

Reasoning

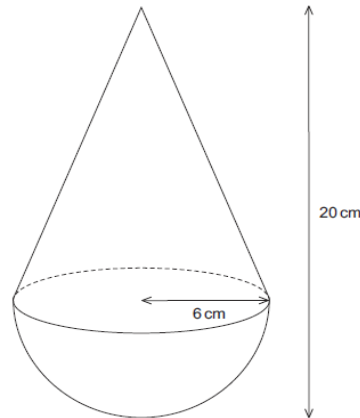
Jamal says "if a cone and a sphere have the same volume and also the same radius, then the height of the cone is bigger than the radius". Is he correct? How much more is the height compared to the radius?

Fluency

Find the volume and surface area of the following shapes:

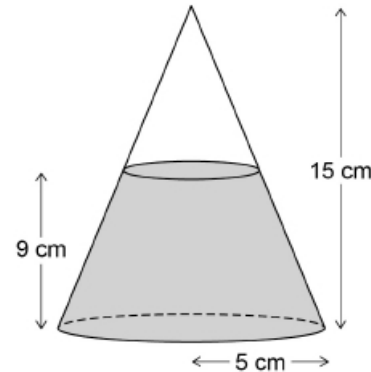
- 1) A sphere with **diameter** of 9m and height m.
- 2) A cone with radius of 3.8cm and height 9cm.

- 3) Calculate the total volume of this 3D shape.



Problem Solving

The conical shaped container is partially filled with water, so that the water has depth of 9cm. What is the volume of the water in the container?



Theta Unit 14: Real Life Graphs

Prior Knowledge

Plot coordinates in all 4 quadrants.

Find the mid-point of a line segment.

Use Pythagoras on right-angle triangles.

Use a conversion graph.

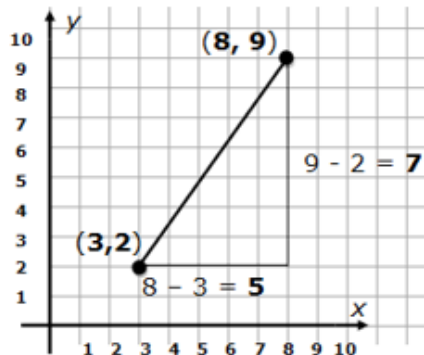
Draw and interpret distance-time graphs.

Distance Between Two Points

The distance between two points, can be seen as a right angle triangle, so we can use Pythagoras to find the distance between two points.

E.G. Find the distance between the points (3,2) and (8,9).

$$\sqrt{7^2 + 5^2} = \sqrt{74} = 8.60232 \dots$$



Real Life Graphs

Example

The graph shows the cost of hiring a chainsaw from saws r us. The company charge a fixed charge plus a daily charge.

Calculate the fixed and daily charge.

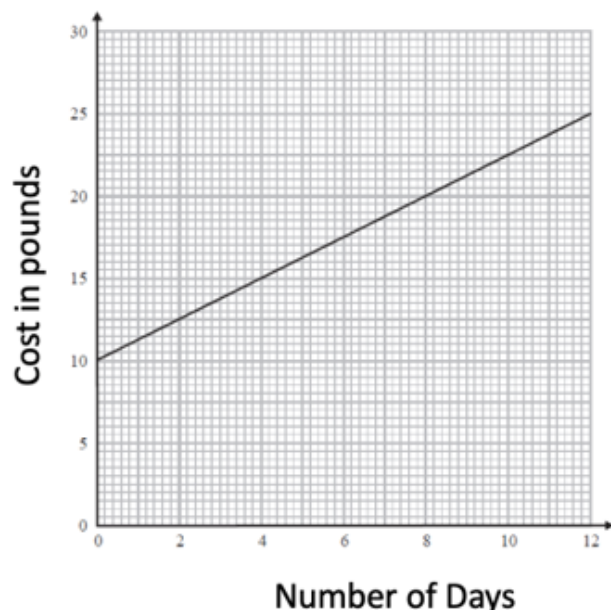
Answer:

Fixed Charge = Starting price for 0 days the intercept, this is £10.

Daily charge = This can be calculated by working out the difference from one day to the next. Day 0 = £10, Day 2 = £12.50.

$$12.50 - 10 = 2.50 \quad 2.50 \div 2 = \text{£}1.25$$

Daily charge = £1.25.

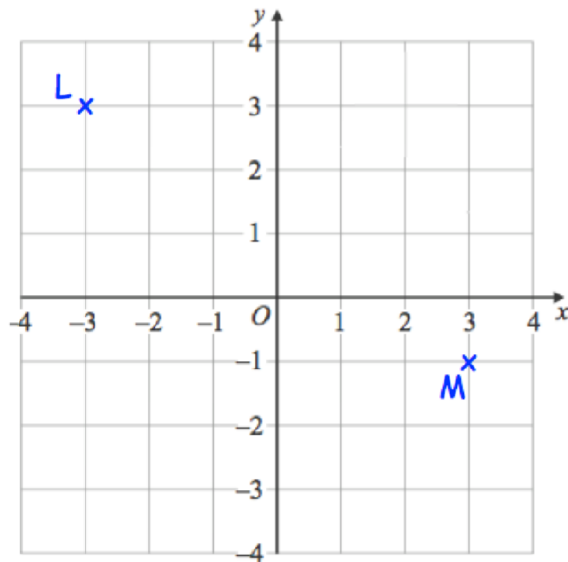


Literacy

Write the definition of gradient.

Define the word intercept.

Problem Solving

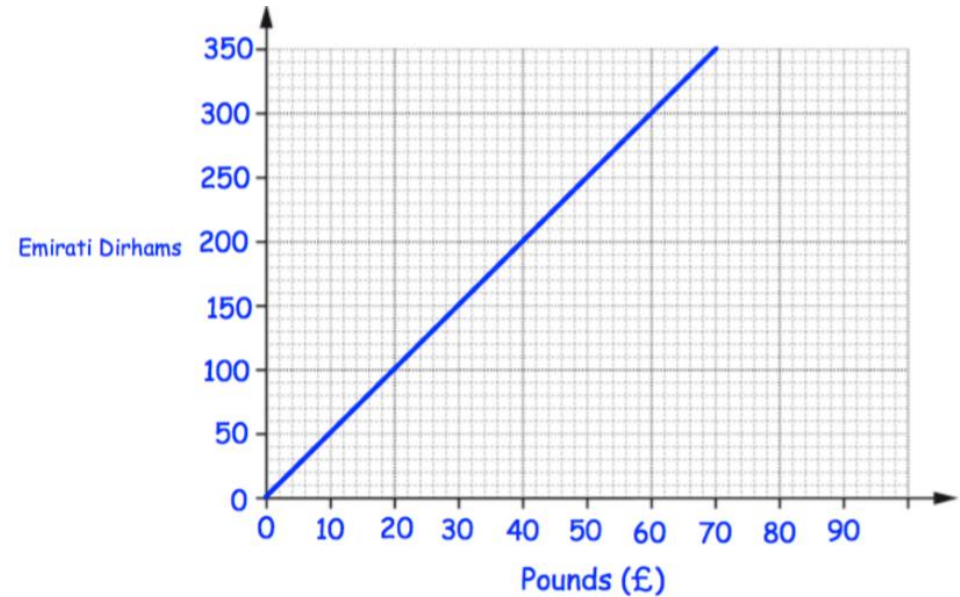


Write down the coordinates of L.

Write down the coordinates of M.

Find the coordinates of the midpoint of LM.

Fluency Reasoning



(a) Convert £50 into Dirhams.

.....Dirhams
(1)

(b) Convert 175 Dirhams into Pounds (£).

£.....
(1)

Tom wants to buy a camera.
In London the camera costs £380.
In Abu Dhabi the camera costs 2000 Dirhams.

In which city is the camera cheaper and by how much?
Give your answer in pounds.

Theta Unit 15: Constructions and Loci

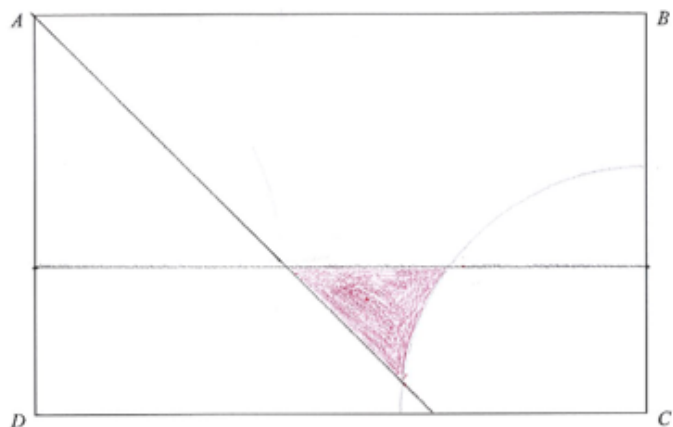
Prior Knowledge

Using a compass. Using a protractor. Draw and use Loci. Draw plans and elevations of 3D shapes.
Construct a perpendicular bisector. Construct an angle bisector. Estimate lengths from scale diagrams.
Measure and use bearings. Know that the perpendicular from a point to a line is the shortest distance to the line.
Draw triangles accurately using a protractor and compass.

Using Loci to Find Regions on Scale Diagrams

Jane wants to plant a tree in the garden. It needs to be at least 5m from C. Nearer to AB than AD and less than 3m from DC.

On the diagram, shade the region where Jane should plant the tree.



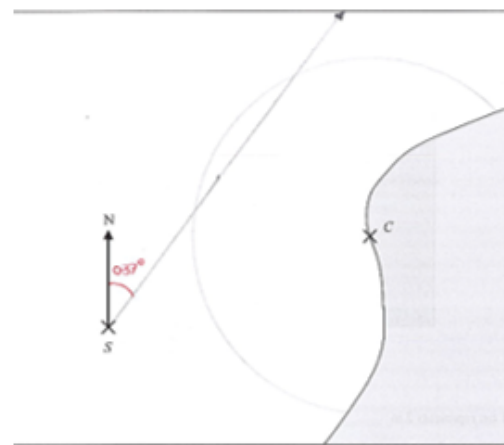
Arc radius around C.
Bisector of AB and AD to find points nearer AB than AD.
Line as a loci from DC to show close enough to DC.
Region shaded red.

Bearings and Loci

Here is a map. S is the position of a ship. C is a point on the coast.

Ships must not sail within 500m of C.
The ship is on a bearing of 037° .

Will the ship sail closer than 500m of C?



Circle around C to represent the area close enough to the ship.

Then the bearing is shown as well.

Yes, as the ship's course does intersect the region.

Literacy

Write the definition of loci.

Use the word loci within a sentence.

Reasoning

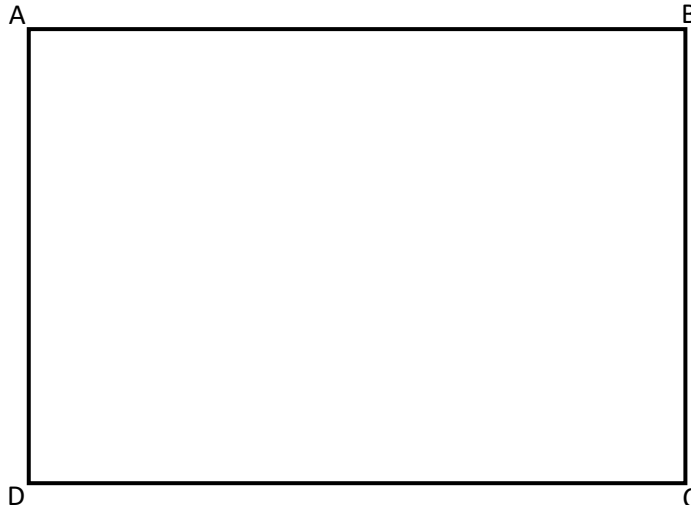
Nathan was asked to show all the points equidistant from A and B. Comment on his response.



Fluency

Find the following bearings

- 1) The bearing of A from B is 138° . What is the bearing of B from A?
- 2) The bearing of C from D is 284° . What is the bearing of D from C?
- 3) The bearing of E from F is 082° . What is the bearing of F from E?



Label the region R,
that is:

Closer to AB than CD

Closer to CD than AD

More than 3cm from C

Problem Solving

Ship A sails on a bearing of 060° at 25km an hour.
Ship B sails on a bearing of 285° at 40km an hour.
If both boats set off at the same time, how far apart are they after 4 hours? Use a scale of 1cm = 30km

